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Anomaly-matching and Higgs-less effective theories

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Abstract: We reconsider the low-energy effective theory for Higgs-less electroweak symmetry breaking: we study the anomaly-matching in the situation where all Goldstone fields disappear from the spectrum as a result of the Higgs mechanism. We find that the global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ symmetry of the underlying theory, which is spontaneously broken $\mathrm{SU}(2)_{L+R} \times \mathrm{U}(1)_{B-L}$, has to be anomaly-free. For the sake of generality, we include the possibility of light spin-1/2 bound states resulting from the dynamics of the strongly-interacting symmetry-breaking sector, in addition to the Goldstone bosons. Such composite fermions may have non-standard couplings at the leading order, and an arbitrary total $B-L$ charge. In order to perform the anomaly-matching in that case, we generalize the construction of the Wess-Zumino effective lagrangian. Composite fermions beyond the three known generations are theoretically allowed, and there are no restrictions from the anomaly-matching on their couplings nor on their $\mathrm{U}(1)_{B-L}$ charge. Absence of global anomalies for the composite sector as a whole does not preclude anomalous triple gauge boson couplings arising from composite fermion triangular diagrams. On the other hand, the trace of $B-L$ over elementary fermions must vanish if all Goldstone modes are to disappear from the spectrum.

Keywords: Spontaneous Symmetry Breaking, Anomalies in Field and String Theories, Beyond Standard Model, Chiral Lagrangians.

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1. Introduction

The detailed dynamics of the electroweak symmetry breaking (EWSB) is so far unknown, and the possibility that it may be strongly-interacting above a few TeV scale is not ruled out. Below that scale a systematic description would then be provided by a Low-Energy Effective Theory (LEET) following the lines initiated by Weinberg [1]. In the limit of small momenta the effective theory remains weakly coupled and the relevant degrees of freedom remain light due to the underlying chiral and gauge symmetries. The LEET is formulated as a systematic expansion in powers of momenta. The list of light fields relevant at low energies whose mass is protected by a symmetry is then a required input. Here, we will assume a minimal content of the composite EWSB sector: the only light bosonic fields are the three Goldstone modes arising from the spontaneous breakdown of the symmetry $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1) \longrightarrow \mathrm{SU}(2)_{L+R} \times \mathrm{U}(1)$, which is supposed to characterize the strongly interacting symmetry breaking sector at low energies (the underlying theory will be referred to as 'techni-theory'). In addition we introduce elementary fields external to the techni-theory: $\mathrm{SU}(2) \times \mathrm{SU}(2)$ Yang-Mills fields and the elementary fermion doublets. Once the elementary sector and the composite sectors are coupled, the three Goldstone bosons produce the masses of the $W^{\pm}$ and Z^0 in the usual way. This scenario corresponds to the case of Higgs-less symmetry breaking, in which no light scalar remains in the spectrum.

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Renormalization is then performed order-by-order in the low-energy expansion. Similarly, unitarity is gradually restored in the same momentum expansion, rather than in powers of coupling constants as would be the case for a renormalizable theory. In a LEET, even though observables do not depend on a cut-off or on a renormalization scale, there is an energy scale inherent to the theory above which the low-energy expansion becomes unreliable. In the case of Chiral Perturbation Theory (χPT) [2, 3], this scale is known to be given by $\Lambda \simeq 4\pi f$ [4], where f is the pion decay constant, although one may hope that the inclusion of additional states (resonances) in the LEET can push this scale up. In the case of EWSB, a similar reasoning leads to an estimate of $\Lambda \simeq 3$ TeV .

At a given level of accuracy, $\mathcal{O}(p^2)$, $\mathcal{O}(p^4)$,..., new renormalized low-energy constants appear in addition to loop contributions. Precision experiments can then be used to fix these constants, and this has been partially done previously, see e.g. [5, 6]. The values of these constants reflect the details of the unknown dynamics of the techni-theory. In the past, different models for the techni-theory and for Higgs-less EWSB have been considered: for instance technicolor models [7], directly inspired by QCD, suggest values of $\mathcal{O}(p^4)$ low-energy constants, which are at variance with electroweak precision tests [8]–[12]. For a more recent discussion, see also [13]. More refined versions such as walking technicolor [14]–[18], are not yet completely excluded [11, 12]. Another particular example is the heavy Higgs limit of the Standard Model (SM) [19, 20, 21, 13]. More recently, Higgs-less models [22, 23, 24] have been revived, in connection with five-dimensional set-ups. The variety of this (incomplete) list incites us to approach the question in a model-independent way within a LEET, trying to understand the generic features of Higgs-less EWSB.

In the Higgs-less LEET, difficulties arise first at the leading $\mathcal{O}(p^2)$ order: given the standard power counting and the symmetry of the problem $\mathrm{SU}(2) \times \mathrm{U}(1)$, one finds $\mathcal{O}(p^2)$ operators, which are not observed experimentally. The problem is then to suppress these unwanted terms naturally. A systematic framework for the resolution of this problem has been proposed in [25]: one appeals to a larger symmetry, characteristic of the situation where the elementary gauge and elementary fermion sector on one hand and the composite symmetry-breaking sector on the other hand are decoupled. This larger natural symmetry is $[\mathrm{SU}(2) \times \mathrm{SU}(2)]^2 \times \mathrm{U}(1)$. The link between the two sectors is introduced via non-propagating spurion fields, which are assumed to be covariantly constant. The solution of these constraints is invariant under a reduced symmetry $\mathrm{SU}(2) \times \mathrm{U}(1)$, introducing the coupling between composite Goldstone modes and elementary fields. Using the relics of the spurions as small expansion parameters, the scheme gives a consistent description for the suppression of the aforementioned unwanted terms, namely non-standard couplings of fermions to vector bosons, and tree-level S parameter. In addition, it allows for a description of fermion masses including mass-splittings within doublets at the same order as the masses themselves, standard CKM mixing and CP violation.

In this paper, we address a second potential problem of Higgs-less effective theories concerning anomaly-matching [26], which requires clarification. The anomalous Ward identities for the symmetry currents of the techni-theory have to be reproduced in the LEET by light spin-0 or spin-1/2 physical states [27]. In the absence of the latter, this amounts to the construction of a Wess-Zumino effective lagrangian involving the Goldstone fields [28].

It is not a priori clear how the anomaly-matching proceeds if all spin-0 physical states disappear from the spectrum due to the Higgs mechanism. Therefore, we consider the possibility of light composite fermions as bound states of the techni-theory and investigate the anomaly-matching condition in a Higgs-less theory with this more general setting. Note that the distinction between composite and elementary fermions from the point of view of the underlying theory is clear: the first ones are not fundamental variables, but appear among the bound states as a consequence of the strong dynamics of the techni-theory, while the latter are introduced as fundamental fields external to the techni-theory. Therefore, elementary fermions do not participate in the anomaly-matching, given that they are described by the same fields both in the low and high-energy descriptions.

The paper is organized as follows.

In section 2, we give an overview of the content of the composite and elementary sectors, and their coupling via spurions. We show that, contrary to elementary fermions, composite fermions have non-standard couplings at $\mathcal{O}(p^2)$.

Section 3 deals with the anomaly-matching between the techni-theory and its effective theory in the general case: we first study the variation of the generating functional for the Noether currents of the techni-theory. This variation is reproduced by a standard Wess-Zumino lagrangian in the absence of composite fermions. We show that, if such fermions are present, it is always possible to build a generalized Wess-Zumino action such that the sum of its variation and the variation of the composite fermion determinant reproduces the anomalies of the techni-theory, whatever the values of the non-standard couplings of the composite fermions. Therefore, the couplings of these composite fermions are unconstrained, including the value of their $\mathrm{U}(1)_{B-L}$ charge: there is no such constraint as the vanishing of the trace of $B-L$ over composite fermions.

The main result of the anomaly-matching in Higgs-less theories is presented in section 4. It concerns the techni-theory itself: its low-energy symmetry $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ must be anomaly-free if all Goldstone modes are to disappear from the spectrum via the Higgs mechanism. This statement is independent of the presence of composite fermions. The vanishing of the trace of $B-L$ for elementary fermions is still required in order to allow for a consistent formulation of the Higgs-less EWSB. Notice that the logical status of this result is quite different from what happens in the SM, where it stems from the requirement of renormalizability.

Finally, in section 5, we give the appropriate formulation for the theory in the unitary gauge. In particular, we study the contribution of triangular diagrams involving composite fermions to triple gauge boson vertices.

We conclude in section 6 by summarizing our main results and mentioning open questions.

2. Composite and elementary sectors

In this section we describe the LEET at lowest order, distinguishing between the composite sector resulting from the dynamics of the techni-theory and the elementary sector. In the bosonic sector, only the four elementary gauge fields that become the $W^{\pm}$, Z^0 and photon are introduced, together with the three real Goldstone bosons (composite fields) required to give them masses. These Goldstone bosons describe the spontaneous breaking of symmetry $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \longrightarrow \mathrm{SU}(2)_{L+R}$ assumed to occur as a result of the strong dynamics of the techni-theory. In addition, we consider the case where the theory has a global vector symmetry $\mathrm{U}(1)_{B-L}$, which is required for our construction.[1]

The generating functional for the seven Noether currents $J_L^{a\mu}, J_R^{a\mu}$, for $a = 1,2,3$, and J^μ_{B-L} that correspond to global symmetries of the techni-theory can be formally defined through the relation

$$\mathrm{e}^{\mathrm{i}\Gamma\left[L_\mu, R_\mu, B^0_\mu\right]} = \int \mathrm{d}\,[T]\, \mathrm{e}^{\mathrm{i}\int \mathrm{d}x \mathcal{L}_{\mathrm{TT}}[T] + J_L^{a\mu}[T] L^a_\mu + J_R^{a\mu}[T] R^a_\mu + J^\mu_{B-L}[T] B^0_\mu}\,. \tag{2.1}$$

Here T collectively denotes the fundamental fields of the techni-theory and $\mathcal{L}_{\mathrm{TT}}$ is its — unspecified — lagrangian. The sources $L^a_\mu, R^a_\mu, B^0_\mu$ in principle allow us to extract Green's functions for the corresponding Noether currents. In fact, the reason we resort to a LEET stems from our ignorance: we do not know what the appropriate variables T are. Instead, we work with the representation of the techni-theory in terms of the low-energy variables, defined as an expansion in powers of momenta and other appropriate parameters [1]. This expansion relies on the symmetries of the theory and the fact that the currents have to satisfy both normal and anomalous Ward identities. The latter are known up to a multiplicative constant, which reflects the field content of the techni-theory at higher energies. The anomalous Ward identities obtained from the fundamental theory have to be reproduced by the LEET itself. Before we turn to this anomaly-matching, we describe the low-energy representation of the symmetry-breaking sector, and then the elementary sector, which is external to the techni-theory. We also briefly review how the coupling between the two sectors is introduced via spurions.

2.1 Low-energy representation of the symmetry-breaking sector

The minimal field content of the symmetry-breaking sector is a triplet of Goldstone bosons. As schematized in figure 1, the Goldstone modes are parameterized in the LEET by an $\mathrm{SU}(2)$ matrix Σ transforming as

$$\Sigma \longmapsto L\Sigma R^\dagger\,, \tag{2.2}$$

with $(L,R) \in \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. In order to deal with the Noether currents generated by the symmetries of the techni-theory, we consider local $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ transformations and introduce the same sources first used in (2.1) as connections in the covariant derivative [2]

$$D_\mu \Sigma = \partial_\mu \Sigma - \mathrm{i} L_\mu \Sigma + \mathrm{i} \Sigma R_\mu\,. \tag{2.3}$$

Up to anomalies, the lagrangian must be invariant if we simultaneously perform the following transformations on the sources [29]

$$L_\mu \longmapsto L L_\mu L^\dagger + \mathrm{i} L \partial_\mu L^\dagger\,, \tag{2.4}$$

[1]In fact, the identification of the $B-L$ transformation with the third component of the right isospin (to obtain the hypercharge transformation) will be obtained *via* the constraint applied on the spurion ϕ. This procedure allows us to maintain the SU(2) custodial symmetry in the lagrangian.

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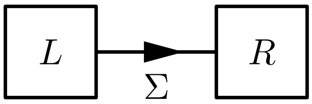

Figure 1: Transformation properties of the Goldstone bosons.

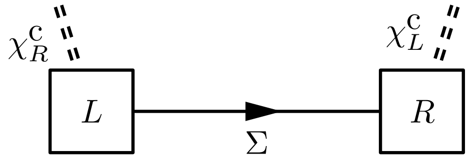

Figure 2: Transformation properties of the composite fermions.

$$R_\mu \longmapsto RR_\mu R^\dagger + \mathrm{i}R\partial_\mu R^\dagger \,. \tag{2.5}$$

The power counting for the low-energy expansion is known [2]

$$\partial_\mu, L_\mu, R_\mu, B^0_\mu = \mathcal{O}(p^1)\,, \tag{2.6}$$

$$f, \Sigma = \mathcal{O}(p^0)\,, \tag{2.7}$$

where f is the Goldstone boson decay constant. The energy scale Λ to which the momenta have to be compared to is known to be approximately equal to $4\pi f$ [4]. The leading-order $\mathcal{O}(p^2)$ lagrangian is then

$$\mathcal{L}_{\mathrm{SB}} = \frac{f^2}{4}\left\langle D_\mu \Sigma D^\mu \Sigma^\dagger \right\rangle, \tag{2.8}$$

where $\langle\cdots\rangle$ denotes the trace of a two-by-two matrix. The full effective lagrangian is to be constructed as the most general expression satisfying the symmetry requirements, organized as an order-by-order expansion in powers of momenta.

It is an open possibility, though not a necessity, that the techni-theory also produces light spin-1/2 bound states: composite fermions. These would then transform under the same $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ transformation as the Goldstone bosons, which are the other bound states of the techni-theory. By including composite fermions in the LEET, we are implicitly assuming that they are light with respect to the scale Λ. This can be understood for instance if they play a role in the anomaly-matching [26]. Consequently, light composite fermions have to be included in a discussion of anomaly-matching for the sake of completeness.

The left and right-handed composite fermion doublets are

$$\chi^{\mathrm{c}}_{L,R} = \frac{1 \mp \gamma_5}{2}\chi^{\mathrm{c}}\,, \tag{2.9}$$

with the appropriate counting given by [30, 13]

$$\chi^{\mathrm{c}} = \mathcal{O}(p^{1/2})\,. \tag{2.10}$$

These fermions are taken to have identical charges under the $\mathrm{U}(1)_{B-L}$ symmetry[2]

$$\chi^{\mathrm{c}}_L \longmapsto {}^t\chi^{\mathrm{c}}_L = R\mathrm{e}^{-\mathrm{i}\frac{B-L}{2}\alpha^0}\chi^{\mathrm{c}}_L\,, \tag{2.11}$$

[2]In order to conform to the standard notation dealing with anomalies, we will call α^0 the gauge function instead of β^0 used in [25].

$$\chi^{\rm c}_R \longmapsto {}^t\chi^{\rm c}_R = L{\rm e}^{-{\rm i}\frac{B-L}{2}\alpha^0}\chi^{\rm c}_R\,. \tag{2.12}$$

Here, $B-L$ is just a generic parameter characterizing each composite fermion doublet.

Except for the $\mathrm{U}(1)_{B-L}$ symmetry, the transformation properties are summarized in figure 2. We will mention briefly at the end of section 3.3 why other possibilities for the transformation properties of these fermions are not of interest in the context of this paper, in connection with the anomaly-matching. The most general lagrangian of order $\mathcal{O}(p^2)$ involving composite fermions and invariant under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ is

$$\begin{aligned}\mathcal{L}_{\rm comp\,.\,ferm\,.} = {} & {\rm i}\overline{\chi^{\rm c}}\gamma^\mu D_\mu\chi^{\rm c} + {\rm i}\delta_L\overline{\chi^{\rm c}_L}\left(\Sigma^\dagger D_\mu\Sigma\right)\chi^{\rm c}_L + \\ & + {\rm i}\delta_R\overline{\chi^{\rm c}_R}\left(\Sigma D_\mu\Sigma^\dagger\right)\chi^{\rm c}_R + \text{four-fermion interactions}\,,\end{aligned} \tag{2.13}$$

where the four-fermion interactions are suppressed by an unknown scale, which has no obvious relation to the scale Λ introduced above in connection with Goldstone fields. Consequently, we will neglect these four-fermion interactions in the sequel. The covariant derivative in (2.13) is given by

$$D_\mu\chi^{\rm c} = \partial_\mu\chi^{\rm c} - {\rm i}\left\{\frac{B-L}{2}B^0_\mu + \frac{1-\gamma_5}{2}R_\mu + \frac{1+\gamma_5}{2}L_\mu\right\}\chi^{\rm c}\,, \tag{2.14}$$

where the $\mathrm{U}(1)_{B-L}$ connection occurring in this covariant derivative transforms as follows

$$B^0_\mu \longmapsto B^0_\mu - \partial_\mu\alpha^0\,. \tag{2.15}$$

Since composite fermions are directly coupled to the Goldstone bosons, a gauge invariant mass term can be included in $\mathcal{L}_{\rm comp\,.\,ferm\,.}$ (2.13)

$$\mathcal{L}_{\rm comp\,.\,ferm\,.} \longmapsto \mathcal{L}_{\rm comp\,.\,ferm\,.} - m\left(\overline{\chi^{\rm c}_L}\Sigma^\dagger\chi^{\rm c}_R + \overline{\chi^{\rm c}_R}\Sigma\chi^{\rm c}_L\right). \tag{2.16}$$

There is nothing, in what we have described up to now, that forces the coefficient m appearing in front of this operator to be small. Notice however that, in order for this last term in (2.16) to be counted as $\mathcal{O}(p^2)$ together with the kinetic term in (2.13) so as to reproduce the pole in the propagator, m should be counted as $\mathcal{O}(p^1)$. This is related to the need for a mechanism to keep this mass light: we are assuming that such a mechanism is at play, and therefore use the counting

$$m = \mathcal{O}(p^1)\,. \tag{2.17}$$

At this level, mass-splittings within doublets are forbidden by the symmetry $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$.

Non-standard couplings δ_L, δ_R, whose strength are not fixed [31, 32], are already present at leading-order in (2.13), due to the symmetry being realized non-linearly on the Goldstone fields.[3] For elementary fermions, such non-standard couplings are suppressed since they necessarily involve spurions, see [25].

[3]In the standard elementary Higgs model, these non-standard terms would appear as dimension six operator.

This completes for the time being the effective low-energy description of the composite sector: the low-energy representation for the generating functional (2.1) is then

$$\mathrm{e}^{\mathrm{i}\Gamma\left[L_\mu,R_\mu,B^0_\mu\right]} = \int \mathrm{d}\left[\Sigma\right]\mathrm{d}\left[\chi^{\mathrm{c}}\right]\mathrm{e}^{\mathrm{i}\int \mathrm{d}x \mathcal{L}_{\mathrm{SB}}\left[\Sigma,L_\mu,R_\mu,B^0_\mu\right]+\mathcal{L}_{\mathrm{comp\,.\,ferm\,.}}\left[\chi^{\mathrm{c}},\Sigma,L_\mu,R_\mu,B^0_\mu\right]+\mathcal{O}(p^4)}\,. \tag{2.18}$$

We now turn to the elementary sector, and to the coupling between the two sectors via spurions.

2.2 The elementary sector and spurion-induced couplings

The elementary sector is external to the techni-theory: in the low-energy description, this results in a separation between the composite sector described in section 2.1, and the elementary sector. The two sectors will only be coupled via spurions.

Among the elementary fields are the gauge fields, which we introduce as an $\mathrm{SU}(2)_{G_0}\times \mathrm{SU}(2)_{G_1}$ Yang-Mills theory, with the connections transforming as

$$G_{0\mu} \longmapsto G_0 G_{0\mu} G_0^\dagger + \frac{\mathrm{i}}{g_0} G_0 \partial_\mu G_0^\dagger\,, \tag{2.19}$$

$$G_{1\mu} \longmapsto G_1 G_{1\mu} G_1^\dagger + \frac{\mathrm{i}}{g_1} G_1 \partial_\mu G_1^\dagger\,, \tag{2.20}$$

where $(G_0, G_1) \in \mathrm{SU}(2)^2$. With the usual power counting [33, 30]

$$G_{0\mu}, G_{1\mu} = \mathcal{O}\left(p^0\right), \tag{2.21}$$

$$g_0, g_1 = \mathcal{O}(p^1)\,, \tag{2.22}$$

the $\mathcal{O}(p^2)$ lagrangian is then simply

$$\mathcal{L}_{\mathrm{gauge}} = -\frac{1}{2}\left\langle G_{1\mu\nu}G_1^{\mu\nu}\right\rangle - \frac{1}{2}\left\langle G_{0\mu\nu}G_0^{\mu\nu}\right\rangle\,. \tag{2.23}$$

In the above, the field-strengths are as usual

$$G_{k\mu\nu} = \partial_\mu G_{k\mu} - \partial_\nu G_{k\mu} - \mathrm{i} g_k \left[G_{k\mu}, G_{k\nu}\right], \qquad \text{for } k = 0, 1\,. \tag{2.24}$$

Elementary fermions doublets χ^{e} are directly coupled to the gauge fields of $\mathrm{SU}(2)_{G_0}\times \mathrm{SU}\left(2\right)_{G_1}$, rather than being charged under the symmetries of the techni-theory $\mathrm{SU}(2)_L \times \mathrm{SU}\left(2\right)_R$ as is the case for composite fermions χ^c (see section 2.1). The elementary fermions transform as

$$\chi^{\mathrm{e}}_L \longmapsto G_1 \mathrm{e}^{-\mathrm{i}\frac{B-L}{2}\alpha^0}\chi^{\mathrm{e}}_L\,, \tag{2.25}$$

$$\chi^{\mathrm{e}}_R \longmapsto G_0 \mathrm{e}^{-\mathrm{i}\frac{B-L}{2}\alpha^0}\chi^{\mathrm{e}}_R\,. \tag{2.26}$$

Note that the value of $B-L$ for elementary fermion and composite fermion doublets are a priori different. The complete $\mathcal{O}(p^2)$ lagrangian describing elementary fermions reads

$$\mathcal{L}_{\mathrm{elem\,.\,ferm\,.}} = \mathrm{i}\overline{\chi^{\mathrm{e}}}\gamma^\mu D_\mu \chi^{\mathrm{e}} + \text{four-fermion interactions}\,, \tag{2.27}$$

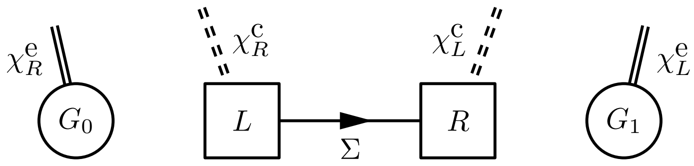

Figure 3: The composite and elementary sectors decoupled.

where

$$D_\mu \chi^{\rm e} = \partial_\mu \chi^{\rm e} - {\rm i} \left\{ \frac{B-L}{2} B_\mu^0 + \frac{1-\gamma_5}{2} g_1 G_{1\mu} + \frac{1+\gamma_5}{2} g_0 G_{0\mu} \right\} \chi^{\rm e} \,. \tag{2.28}$$

Here again, it is assumed that the four-fermion interactions are suppressed by another sufficiently large scale, and are henceforth neglected. At this stage, there are no couplings between the composite and elementary sectors, as evidenced in figure 3, where the transformation properties of all fields introduced up to now are summarized. The full $\mathcal{O}(p^2)$ lagrangian is indeed the sum of terms involving composite fields on one side and elementary ones on the other

$$\mathcal{L}_2 = (\mathcal{L}_{\rm SB} + \mathcal{L}_{\rm comp\,.\,ferm\,.}) + (\mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm elem\,.\,ferm\,.}) \,. \tag{2.29}$$

Its invariance group is

$$S_{\rm natural} = {\rm SU}(2)_L \times {\rm SU}(2)_R \times {\rm SU}(2)_{G_0} \times {\rm SU}(2)_{G_1} \times {\rm U}(1)_{B-L} \,. \tag{2.30}$$

The couplings between the Goldstone modes and the elementary gauge fields are introduced through constraints enforced via spurions, following the formalism developed at length in [25]. For convenience, we reproduce here the main steps. The spurions link the ${\rm SU}(2)_L \times {\rm SU}(2)_R \times {\rm U}(1)_{B-L}$ transformations on one side with the ${\rm SU}(2)_{G_0} \times {\rm SU}(2)_{G_1}$ gauge group on the other side. We introduce the two-by-two matrices X and $\tilde{Y}$ and the complex doublet ϕ. X is taken to satisfy the reality condition

$$X = \tau^2 X^* \tau^2 \,. \tag{2.31}$$

The transformation properties of the spurions are

$$X \longmapsto R X G_1^\dagger \,, \tag{2.32}$$

$$\tilde{Y} \longmapsto G_0 \tilde{Y} L^\dagger \,, \tag{2.33}$$

$$\phi \longmapsto G_0 {\rm e}^{{\rm i}\frac{\alpha^0}{2}} \phi \,. \tag{2.34}$$

Conditions of covariant constancy

$$D_\mu X = \partial_\mu X - {\rm i} R_\mu X + {\rm i} g_1 X G_{1\mu} = 0 \,, \tag{2.35}$$

$$D_\mu \tilde{Y} = \partial_\mu \tilde{Y} - {\rm i} g_0 G_{0\mu} \tilde{Y} + {\rm i} \tilde{Y} L_\mu = 0 \,, \tag{2.36}$$

$$D_\mu \phi = \partial_\mu \phi - {\rm i} g_0 G_{0\mu} \phi + {\rm i} \frac{B_\mu^0}{2} \phi = 0 \,, \tag{2.37}$$

are imposed on the spurions. These constraints imply the correct vacuum alignment of gauge connections and the coupling of gauge fields with Goldstone bosons. The solution

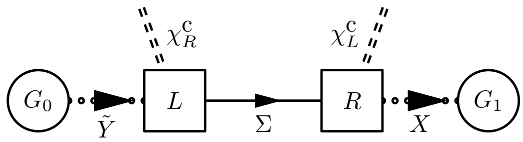

Figure 4: Spurion couplings in the presence of composite fermions.

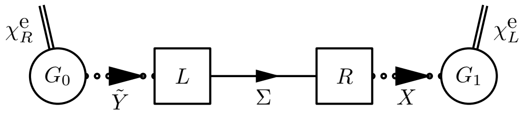

Figure 5: Spurion couplings in the presence of elementary fermions.

to the constraints (2.35)–(2.37) was given in [25]: it is possible to find a gauge — which we will call *standard gauge* — where the spurions reduce to four real constants and one phase φ, yielding

$$X|_{\text{s.g.}} = \xi \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{2.38}$$

$$\tilde{Y}\Big|_{\text{s.g.}} = \mathrm{e}^{\mathrm{i}\varphi} \begin{pmatrix} \eta_1 & 0 \\ 0 & \eta_2 \end{pmatrix}, \tag{2.39}$$

$$\phi|_{\text{s.g.}} = \begin{pmatrix} \zeta \\ 0 \end{pmatrix}. \tag{2.40}$$

The constants ξ, η_1, η_2 and ζ will be considered as small parameters. For notation purposes, we introduce the power counting

$$\xi, \eta_1, \eta_2 = \mathcal{O}(\epsilon)\,. \tag{2.41}$$

The spurion ζ is likely to be even smaller [25] and we count it separately. In the sequel, the phase φ does not play any physical role: it can be eliminated by appropriate redefinitions.

The standard gauge in which (2.38)–(2.40) hold is reached by making use of an $\mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ transformation, together with a $\mathrm{SU}(2)_{G_0} \times \mathrm{SU}(2)_L / \mathrm{U}(1)_{L+G_0,\tau^3}$ transformation, leaving us with an $\mathrm{SU}(2)_{G_1} \times \mathrm{U}(1)_{L+G_0,\tau^3}$ gauge freedom. Indeed, in this standard gauge, the connections are identified as a consequence of the constraints (2.35)–(2.37) as follows

$$R_\mu|_{\text{s.g.}} = g_1 G_{1\mu}\,, \tag{2.42}$$

$$L_\mu^{1,2}\Big|_{\text{s.g.}} = G_{0\mu}^{1,2} = 0\,, \tag{2.43}$$

$$L_\mu^3\Big|_{\text{s.g.}} = B_\mu^0\Big|_{\text{s.g.}} = g_0 G_{0\mu}^3 = g_0 b_\mu^0\,. \tag{2.44}$$

In the last equation, a new notation b_μ^0 for the U(1) gauge field has been introduced. In summary, the spurions $\tilde{Y}, X, \phi$ have enabled us to reduce the symmetry from S_{natural} down to

$$S_{\text{reduced}} = \mathrm{SU}(2) \times \mathrm{U}(1)_Y\,. \tag{2.45}$$

S_{reduced} is actually the residual the symmetry of the solution of the constraints (2.35)–(2.37).

The transformation properties of all fields introduced up to now are summarized in figure 4 and 5, where both cases with composite and elementary fermions are presented separately for convenience.

We are now in a position to write down the lowest-order terms in the full effective lagrangian. At leading order $\mathcal{O}(p^2\epsilon^0\zeta^0)$, the lagrangian actually coincides with (2.29). The interactions appear through the constraints (2.35)–(2.37)

$$\mathcal{L}^{(2,0,0)} = (\mathcal{L}_{\mathrm{SB}} + \mathcal{L}_{\mathrm{comp\,.\,ferm\,.}} + \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{elem\,.\,ferm\,.}})|_{\mathrm{constraints}}\,. \tag{2.46}$$

Terms invariant under S_{natural} (2.30) other than those included in (2.46) will involve additional powers of momentum (or coupling constants) or powers of the spurions, and are therefore suppressed. The couplings are best evidenced by writing down the covariant derivative for the Goldstone fields and then injecting the solution to the constraints, given in (2.42)–(2.44). One finds

$$\nabla_\mu \Sigma = D_\mu \Sigma|_{\mathrm{s.g.}} = \partial_\mu \Sigma - \mathrm{i} g_0 b^0_\mu \frac{\tau^3}{2}\Sigma + \mathrm{i} g_1 \Sigma G_{1\mu}\,. \tag{2.47}$$

Performing the same manipulation for the covariant derivatives acting on both composite (2.14) and elementary fermions (2.28), the same expression is obtained

$$\nabla_\mu \chi_L = D_\mu \chi_L|_{\mathrm{s.g.}} = \partial_\mu \chi_L - \mathrm{i}\left(g_1 G_{1\mu} + g_0 \frac{B-L}{2} b^0_\mu\right)\chi_L\,, \tag{2.48}$$

$$\nabla_\mu \chi_R = D_\mu \chi_R|_{\mathrm{s.g.}} = \partial_\mu \chi_R - \mathrm{i} g_0 \left(\frac{\tau^3}{2} + \frac{B-L}{2}\right) b^0_\mu \chi_R\,, \tag{2.49}$$

where we have removed the superscripts e and c. This is indeed the desired result: we recognize SM-like covariant derivatives up to a change in notation. This however does not imply that the couplings of the composite fermions to the elementary gauge fields are identical to those of the SM fermions since the terms appearing with the free multiplicative factors δ_L, δ_R are allowed at leading order for composite fermions (2.13).

Writing the $\mathcal{O}(p^2\epsilon^0\zeta^0)$ lagrangian (2.46) in the same standard gauge where (2.38)–(2.40) hold, we obtain

$$\begin{aligned}
\mathcal{L}^{(2,0,0)}\Big|_{\mathrm{s.g.}} = {} & \frac{f^2}{4}\left\langle \nabla_\mu \Sigma \nabla^\mu \Sigma^\dagger \right\rangle - \frac{1}{2}\langle G_{1\mu\nu} G_1^{\mu\nu}\rangle - \frac{1}{4} b^0_{\mu\nu} b^{0\mu\nu} + \mathrm{i}\overline{\chi^{\mathrm{e}}}\gamma^\mu \nabla_\mu \chi^{\mathrm{e}} + \\
& + \mathrm{i}\overline{\chi^{\mathrm{c}}}\gamma^\mu \nabla_\mu \chi^{\mathrm{c}} - m\left(\overline{\chi^{\mathrm{c}}_L}\Sigma^\dagger \chi^{\mathrm{c}}_R + \overline{\chi^{\mathrm{c}}_R}\Sigma \chi^{\mathrm{c}}_L\right) + \\
& + \delta_L \overline{\chi^{\mathrm{c}}_L}\gamma^\mu \left(\mathrm{i}\Sigma^\dagger \partial_\mu \Sigma + g_0 b^0_\mu \Sigma^\dagger \frac{\tau^3}{2}\Sigma - g_1 G_{1\mu}\right)\chi^{\mathrm{c}}_L + \\
& + \delta_R \overline{\chi^{\mathrm{c}}_R}\gamma^\mu \left(\mathrm{i}\Sigma \partial_\mu \Sigma^\dagger + g_1 \Sigma G_{1\mu}\Sigma^\dagger - g_0 b^0_\mu \frac{\tau^3}{2}\right)\chi^{\mathrm{c}}_R\,,
\end{aligned} \tag{2.50}$$

with $b^0_{\mu\nu}$ a standard U(1) field-strength

$$b^0_{\mu\nu} = \partial_\mu b^0_\nu - \partial_\nu b^0_\mu\,. \tag{2.51}$$

The main specificity of composite fermions at this level comes from the non-standard δ_L, δ_R terms. It would seem that fermions with such couplings are ruled out by experiments testing the universality of left-handed couplings as well as couplings of right-handed fermions to the $W^{\pm}$. However, one may entertain the view that there may be such fermions in addition to the three known generations. They must then be heavier in order to have eluded detection at accelerators until now, and do not necessarily constitute a full generation in the usual sense: the sum of $B-L$ over these composite fermions may be non-vanishing.[4] As it happens, such composite fermions naturally come out heavier than the elementary ones: this is evidenced by the fact that we can build a mass term for them that does not involve spurions, as shown in (2.16), whereas mass terms for elementary fermions would be suppressed by two powers of spurions [25]. On the other hand, mass-splittings appear in both cases only at $\mathcal{O}(p^1\epsilon^2\zeta^0)$ since they require the spurion $\tilde{Y}$, meaning that the ratio of splittings to the mean mass is smaller for composite fermions doublets than for elementary ones. Of course, the two types of fermions may also mix since multiplication of one type of field by the appropriate spurion yields a field transforming as the other type of fermion. Such a mixing is a higher-order effect in the spurion expansion, and is expected to be small. The distinction between elementary and composite fermions therefore holds order-by-order in the spurion expansion.

Except for the non-standard couplings δ_L, δ_R of the composite fermions (or in the absence of the latter), the lagrangian (2.50) leads to EWSB with the same tree-level relations as in the SM without physical Higgs particle [25].

3. Anomalies

In this section, we discuss the anomaly-matching between the LEET and the underlying techni-theory. We perform this analysis in the general case, taking into account composite fermions if present. Since we will be dealing with the matching between the techni-theory and its low-energy representation, the introduction of the gauge symmetry $\mathrm{SU}(2)_{G_0} \times \mathrm{SU}(2)_{G_1}$ can be relegated to a later stage: the corresponding gauge fields only play the role of spectators. The same is true of the spurions. We can thus use a description involving only the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ symmetry: we focus on the relation between the generating functional $\Gamma\left[L_\mu, R_\mu, B^0_\mu\right]$ defined in (2.1) and its expression in terms of the low-energy variables (2.18).

3.1 Anomalies of the techni-theory for low-energy symmetries

The Noether currents of the techni-theory $J^{a\mu}_L, J^{a\mu}_R, J^{\mu}_{B-L}$ were introduced in (2.1): they correspond to the global symmetries of the techni-theory that operate on the low-energy fields. We study their anomalous Green's functions denoting by t a generic transformation in the group of low-energy symmetries of the techni-theory $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$. We parameterize the transformation t by the seven functions $\alpha = \alpha^a\tau^a/2$, $\beta = \beta^a\tau^a/2$ and α^0 as

$$t = \left(L = \mathrm{e}^{\mathrm{i}(\alpha-\beta)}, R = \mathrm{e}^{\mathrm{i}(\alpha+\beta)}, \alpha^0\right). \tag{3.1}$$

Under a transformation t, the sources are modified according to

$$L_\mu \longmapsto {}^tL_\mu = LL_\mu L^\dagger + \mathrm{i}L\partial_\mu L^\dagger\,, \tag{3.2}$$

[4] We will come back to the value of the trace of $B-L$ for elementary fermions in section 4.2.

$$R_\mu \longmapsto {}^tR_\mu = RR_\mu R^\dagger + \mathrm{i}R\partial_\mu R^\dagger\,, \tag{3.3}$$
$$B^0_\mu \longmapsto {}^tB^0_\mu = B^0_\mu - \partial_\mu \alpha^0\,. \tag{3.4}$$

At this point, it is useful to introduce differential operators acting on the sources in order to reproduce the above transformations. However, in addition to these sources, we have introduced the set of Goldstone fields $\{\pi^a|a=1,2,3\}$ in order to reproduce the low-energy singularities of the theory. These fields are collected in the unitary matrix Σ

$$\Sigma = \mathrm{e}^{\mathrm{i}\frac{\pi^a\tau^a}{f}} = \cos(\hat\pi) + \mathrm{i}\frac{\pi}{f\hat\pi}\sin(\hat\pi)\,, \tag{3.5}$$

where we have defined $\pi = \pi^a\tau^a$ and $\hat\pi = \sqrt{\pi^a\pi^a}/f$. We are therefore interested in a definition of the differential operator that also acts directly on these Goldstone fields [28, 34] — as opposed to it operating via the equations of motion [3]. In order to reproduce the transformation of the Goldstone modes (2.2) under t, we introduce the following infinitesimal construction, explicited as the action of $D(\alpha,\beta,\alpha^0)$ on a generic functional $f[\Sigma, L_\mu, R_\mu, B^0_\mu]$

$$\begin{aligned} D(\alpha,\beta,\alpha^0)f[\Sigma,L_\mu,R_\mu,B^0_\mu] =& \\ = \int \mathrm{d}x \Bigg\{ & \alpha^0\partial_\mu\frac{\delta f}{\delta B^0_\mu(x)} + \{-(\alpha^a+\beta^a)\partial_\mu + \mathrm{i}[\alpha+\beta,R_\mu]^a\}\frac{\delta f}{\delta R^a_\mu(x)} + \\ & +\{-(\alpha^a-\beta^a)\partial_\mu + \mathrm{i}[\alpha-\beta,L_\mu]^a\}\frac{\delta f}{\delta L^a_\mu(x)} + \\ & +\Bigg\{\mathrm{i}[\alpha,\pi]^a + \frac{f\hat\pi}{\sin(\hat\pi)}\Bigg(-\beta^a\cos(\hat\pi) + \beta^b\frac{\pi^b\pi^a}{f^2\hat\pi^2}\Bigg(\cos(\hat\pi) - \frac{1}{\hat\pi}\sin(\hat\pi)\Bigg)\Bigg)\Bigg\} \times \\ & \frac{\delta f}{\delta\pi^a(x)}\Bigg\}\,. \end{aligned} \tag{3.6}$$

One can check explicitly that (3.6) indeed yields the desired result

$$\mathrm{e}^{D(\alpha,\beta,\alpha^0)} f\left[\Sigma, L_\mu, R_\mu, B^0_\mu\right] = f\left[{}^t\Sigma, {}^tL_\mu, {}^tR_\mu, {}^tB^0_\mu\right]. \tag{3.7}$$

With these definitions, we can perform the Wess-Zumino construction in our setting: since we will build upon the result in the sequel, we describe this standard procedure. We use the differential operator $D(\alpha,\beta,\alpha^0)$ of (3.6) to express the variation of the generating functional $\Gamma\left[L_\mu, R_\mu, B^0_\mu\right]$ defined in (2.1) under a transformation applied on the techni-fields T. This variation is determined by the content of the techni-theory. Assuming the group of transformation $\mathrm{SU}(2)_L \times \mathrm{SU}\,(2)_R \times \mathrm{U}(1)_{B-L}$ to act linearly on the techni-fermions, which constitute a subset of the fundamental variables T, the action of the differential operator defined in (3.6) will yield a standard Bardeen variation [35, 36, 37, 38, 39, 40, 2, 41]. This variation is therefore known up to a multiplicative coefficient k_{TT}, which constitutes the only unknown here

$$D(\alpha,\beta,\alpha^0)\Gamma\left[L_\mu, R_\mu, B^0_\mu\right] = -k_{\mathrm{TT}}\int \mathrm{d}x\,\left\langle \beta\Omega\left[L_\mu, R_\mu, B^0_\mu\right]\right\rangle\,, \tag{3.8}$$

where a standard Bardeen functional appears

$$\Omega\left[L_\mu, R_\mu, B^0_\mu\right] = \mathcal{O}\left(p^4\epsilon^0\zeta^0\right). \tag{3.9}$$

For the group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$, this functional assumes the form [42]

$$\Omega\left[L_\mu, R_\mu, B_\mu^0\right] = \frac{1}{16\pi^2}\varepsilon^{\mu\nu\rho\sigma}B_{\mu\nu}^0\left(\partial_\rho L_\sigma + \partial_\rho R_\sigma - \mathrm{i}\left[R_\rho, L_\sigma\right]\right). \tag{3.10}$$

For a theory such as QCD, the value of the analogue of k_{TT} is known: it is equal to $N_c/3$ where N_c is the number of colors and the division by three is due to the baryon number of the quarks being equal to $1/3$ with the standard normalization of the $\mathrm{U}(1)_{B-L}$ charges. We will come back to the value of the constant k_{TT} later in section 4.1.

One may check explicitly, using relation (3.7), that the Bardeen functional in (3.10) verifies the Wess-Zumino integrability conditions [28] expressing the commutation relation of the algebra

$$D\left(\frac{\tau^a}{2}, 0, 0\right)\Omega^b\left[L_\mu, R_\mu, B_\mu^0\right] = \varepsilon^{abc}\Omega^c\left[L_\mu, R_\mu, B_\mu^0\right], \tag{3.11}$$

$$\varepsilon^{abc}D\left(0, \frac{\tau^a}{2}, 0\right)\Omega^b\left[L_\mu, R_\mu, B_\mu^0\right] = 0, \tag{3.12}$$

$$D\left(0, 0, \alpha^0\right)\Omega\left[L_\mu, R_\mu, B_\mu^0\right] = 0. \tag{3.13}$$

These consistency conditions imply that the variation (3.8) can be reproduced in the low-energy theory by an $\mathcal{O}(p^4\epsilon^0\zeta^0)$ Wess-Zumino action $S_{\mathrm{WZ}}[\Sigma, L_\mu, R_\mu, B_\mu^0]$ depending on the Goldstone boson fields, such that

$$D\left(\alpha, \beta, \alpha^0\right)S_{\mathrm{WZ}}\left[\Sigma, L_\mu, R_\mu, B_\mu^0\right] = -\int \mathrm{d}x \left\langle \beta\Omega\left[L_\mu, R_\mu, B_\mu^0\right]\right\rangle. \tag{3.14}$$

The appropriate action $S_{\mathrm{WZ}}[\Sigma, L_\mu, R_\mu, B_\mu^0]$ is found by integration of the differential equation (3.14). Under the boundary condition

$$S_{\mathrm{WZ}}\left[\mathbb{1}, L_\mu, R_\mu, B_\mu^0\right] = 0, \tag{3.15}$$

one performs the standard Wess-Zumino construction [28] to obtain[5]

$$\begin{aligned} &S_{\mathrm{WZ}}\left[\Sigma, L_\mu, R_\mu, B_\mu^0\right] \\ &\quad = -\frac{1}{32\pi^2}\int \mathrm{d}x\varepsilon^{\mu\nu\rho\sigma}B_{\mu\nu}^0\Big\langle \Sigma R_\rho\Sigma^\dagger L_\sigma - R_\rho L_\sigma - \mathrm{i}\Sigma^\dagger\left(\partial_\rho\Sigma\right)\Big(R_\sigma + \Sigma^\dagger L_\sigma\Sigma\Big)\Big\rangle - \\ &\quad - \frac{1}{48\pi^2}\int \mathrm{d}x\varepsilon^{\mu\nu\rho\sigma}B_\mu^0\Big\langle \Sigma^\dagger\left(\partial_\nu\Sigma\right)\Sigma^\dagger\left(\partial_\rho\Sigma\right)\Sigma^\dagger\left(\partial_\sigma\Sigma\right)\Big\rangle. \end{aligned} \tag{3.16}$$

From this formula, it can be checked explicitly, using the properties of the differential operator (3.6) that we obtain the desired variation (3.14). Since this variation does not involve the fields π^a, and since the integration measure over the Goldstone bosons is invariant under the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ symmetry, we immediately deduce the variation of the generating functional. We thus get the usual result that this standard Wess-Zumino action, when multiplied by the factor k_{TT} and added to a gauge-invariant action, reproduces the anomalous variation of the generating functional (3.8). This is the standard

[5]In this SU(2) case, there is no need to introduce a fifth compact dimension: the Wess-Zumino action is simply the space-time integral of an ordinary lagrangian.

anomaly-matching procedure. However, we also want to consider the case with fermionic bound-states in the low-energy spectrum: these will in general imply an additional variation of the action under the transformation t. The variation will involve the Goldstone fields themselves and will depend on the $\mathrm{U}(1)_{B-L}$ charges of the composite fermions as well as on their couplings.

3.2 Variation of the determinant for composite fermions

On the LEET side, there is a contribution from composite fermions [26, 43, 44] to the anomalous variation of the generating functional $\Gamma\left[L_\mu, R_\mu, B^0_\mu\right]$ under a transformation t. We consider a doublet of composite fermions χ^{c} as introduced in section 2.1. To make things more compact, we define the operator $\hat{\mathcal{D}}_{\mathrm{c}}$ depending only on seven vector sources by its action on χ^{c} as follows

$$\hat{\mathcal{D}}_{\mathrm{c}}\left[\hat{L}_\mu, \hat{R}_\mu, B^0_\mu\right]\chi^{\mathrm{c}} = \gamma^\mu\left\{\mathrm{i}\partial_\mu + \frac{B-L}{2}B^0_\mu + \frac{1-\gamma_5}{2}\hat{R}_\mu + \frac{1+\gamma_5}{2}\hat{L}_\mu\right\}\chi^{\mathrm{c}}\,. \tag{3.17}$$

This allows us to write the lagrangian for composite fermions as

$$\mathcal{L}_{\mathrm{comp\,.\,ferm\,.}} = \overline{\chi^{\mathrm{c}}}\hat{\mathcal{D}}_{\mathrm{c}}\chi^{\mathrm{c}}\,, \tag{3.18}$$

provided we define the objects with carets by the relations

$$\hat{L}_\mu = L_\mu + \mathrm{i}\delta_R\Sigma D_\mu\Sigma^\dagger\,, \tag{3.19}$$

$$\hat{R}_\mu = R_\mu + \mathrm{i}\delta_L\Sigma^\dagger D_\mu\Sigma\,, \tag{3.20}$$

and we treat the Yukawa term as a perturbation. Given that the transformation properties for the quantities with carets are just the same as for the original ones

$$\hat{L}_\mu \longmapsto {}^t\hat{L}_\mu = L\hat{L}_\mu L^\dagger + \mathrm{i}L\partial_\mu L^\dagger\,, \tag{3.21}$$

$$\hat{R}_\mu \longmapsto {}^t\hat{R}_\mu = R\hat{R}_\mu R^\dagger + \mathrm{i}R\partial_\mu R^\dagger\,, \tag{3.22}$$

one can derive the following expression for the $\mathcal{O}(p^4\epsilon^0\zeta^0)$ variation of the fermion determinant under a generic transformation t

$$-\mathrm{i}D(\alpha,\beta,\alpha^0)\ln\mathrm{Det}\,\hat{\mathcal{D}}_{\mathrm{c}}\left[\hat{L}_\mu, \hat{R}_\mu, B^0_\mu\right] = (B-L)\int\mathrm{d}x\left\langle\beta\Omega\left[\hat{L}_\mu, \hat{R}_\mu, B^0_\mu\right]\right\rangle. \tag{3.23}$$

In this result, the same Bardeen functional as in (3.8) occurs, but with different arguments. It can be rewritten as

$$\Omega\left[\hat{L}_\mu, \hat{R}_\mu, B^0_\mu\right] = \Omega\left[L_\mu, R_\mu, B^0_\mu\right] + \Omega_{\delta_L,\delta_R}\left[\Sigma, L_\mu, R_\mu, B^0_\mu\right], \tag{3.24}$$

where $\Omega_{\delta_L,\delta_R}$ vanishes when both δ_L and δ_R are set equal to zero

$$\begin{aligned}\Omega_{\delta_L,\delta_R}\left[\Sigma, L_\mu, R_\mu, B^0_\mu\right] = \frac{1}{16\pi^2}\varepsilon^{\mu\nu\rho\sigma}B^0_{\mu\nu}\Big\{&\delta_R\left(\mathrm{i}\partial_\rho\left(\Sigma D_\sigma\Sigma^\dagger\right) + \left[R_\rho, \Sigma D_\sigma\Sigma^\dagger\right]\right) + \\ &+\delta_L\left(\mathrm{i}\partial_\rho\left(\Sigma^\dagger D_\sigma\Sigma\right) + \left[L_\rho, \Sigma^\dagger D_\sigma\Sigma\right]\right) +\end{aligned}$$

$$+\,\mathrm{i}\delta_L\delta_R\left[\Sigma^\dagger D_\rho\Sigma, \Sigma D_\sigma\Sigma^\dagger\right]\Big\}\,. \qquad (3.25)$$

One can see, from the transformation properties for $\hat{L}_\mu$ and $\hat{R}_\mu$ given in (3.21) and (3.22), that the Bardeen variation in (3.23) will also satisfy the Wess-Zumino integrability conditions as in (3.11)–(3.13), although this time the conditions themselves involve the direct action of the operator D on the π^a fields. In fact the non-standard variation $\Omega_{\delta_L,\delta_R}[\Sigma, L_\mu, R_\mu, B^0_\mu]$ given in (3.25) satisfies the consistency conditions on its own as well. Note that this remains true whatever the value of the parameters δ_L and δ_R. This means that we will be able to build a non-standard Wess-Zumino lagrangian reproducing the additional non-standard variation $\Omega_{\delta_L,\delta_R}\left[\Sigma, L_\mu, R_\mu, B^0_\mu\right]$ separately, which we will need to do later. One can exhibit the explicit expression for such an $\mathcal{O}(p^4\epsilon^0\zeta^0)$ lagrangian $S_{\delta_L,\delta_R}[\Sigma, L_\mu, R_\mu, B^0_\mu]$ according to

$$\begin{aligned} S_{\delta_L,\delta_R}\left[\Sigma, L_\mu, R_\mu, B^0_\mu\right] = -\frac{1}{32\pi^2}\int \mathrm{d}x\varepsilon^{\mu\nu\rho\sigma}B^0_{\mu\nu}\Big\langle &\mathrm{i}\left(\delta_L L_\rho - \delta_R R_\rho\right)\left(\Sigma^\dagger D_\sigma\Sigma + \Sigma D_\sigma\Sigma^\dagger\right) - \\ &- \left(\delta_L - \delta_R\right)\partial_\rho\Sigma^\dagger D_\sigma\Sigma + \\ &+ \delta_L\delta_R\left(\Sigma^\dagger D_\rho\Sigma\Sigma D_\sigma\Sigma^\dagger - D_\rho\Sigma^\dagger D_\sigma\Sigma\right)\Big\rangle\,, \end{aligned} \qquad (3.26)$$

from which it can be verified explicitly that the appropriate variation is recovered

$$D(\alpha,\beta,\alpha^0)S_{\delta_L,\delta_R}[\Sigma, L_\mu, R_\mu, B^0_\mu] = -\int \mathrm{d}x\langle\beta\Omega_{\delta_L,\delta_R}[\Sigma, L_\mu, R_\mu, B^0_\mu]\rangle\,. \qquad (3.27)$$

Notice that we have chosen S_{δ_L,δ_R} in (3.26) to satisfy the boundary condition

$$S_{\delta_L,\delta_R}\left[\mathbb{1}, L_\mu, R_\mu, B^0_\mu\right] = 0\,. \qquad (3.28)$$

3.3 Anomaly-matching

In section 3.1 we obtained the total anomalous variation of the generating functional $\Gamma\left[L_\mu, R_\mu, B^0_\mu\right]$, as deduced from the assumed properties of the techni-theory. In this same section, we gave the form of the standard Wess-Zumino lagrangian to be added to the invariant effective lagrangian so that the bosonic part of the LEET reproduced the full variation. Then in section 3.2, we considered the case where the LEET contains composite fermions in addition to Goldstone bosons. We then determined the contribution of the variation of the fermion determinant to the anomalous variation of the lagrangian (since this variation in general contains Σ, and hence an integration over Σ still has to be performed to obtain the action). In that case, on the LEET side, it is the combination of the bosonic variation (the variation of a Wess-Zumino-like term to be determined) and of the variation of the fermion determinant (3.23), which has to match the total variation derived from the techni-theory (3.8). Finding the appropriate lagrangian for the LEET is indeed what the anomaly-matching is about. In our case, this means determining whether there are restrictions on the constants δ_L, δ_R and $B-L$ for the matching to be possible, and finding the form of the Wess-Zumino-like lagrangian in the bosonic sector: we will perform this matching at $\mathcal{O}(p^4\epsilon^0\zeta^0)$, where the question first arises.

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With this in mind, we now consider how the variation of $\Gamma\left[L_\mu, R_\mu, B^0_\mu\right]$ is reproduced in the effective theory. Integrating over the composite fermions in (2.18), the low-energy representation for the generating functional $\Gamma[L_\mu, R_\mu, B^0_\mu]$ takes the form

$$\mathrm{e}^{\mathrm{i}\Gamma\left[L_\mu,R_\mu,B^0_\mu\right]} = \int \mathrm{d}\left[\Sigma\right] \mathrm{e}^{\mathrm{i}S_{\mathrm{SB}}\left[\Sigma,L_\mu,R_\mu,B^0_\mu\right]+\ln\mathrm{Det}\,\hat{\mathcal{D}}_{\mathrm{c}}\left[\hat{L}_\mu,\hat{R}_\mu,B^0_\mu\right]+\mathcal{O}(p^4)}\,, \tag{3.29}$$

with the obvious definition $S_{\mathrm{SB}} = \int \mathrm{d}x\mathcal{L}_{\mathrm{SB}}$. The variation of $\Gamma\left[L_\mu, R_\mu, B^0_\mu\right]$ given by expression (3.29) has to equal the one given in (3.8). This is the anomaly-matching condition of 't Hooft [26], rephrased in the language of effective theories.

Focusing on the variation of the generating functional as reproduced by the LEET (3.29), we use successive rewritings to obtain

$$\mathrm{e}^{D(\alpha,\beta,\alpha^0)}\mathrm{e}^{\mathrm{i}\Gamma\left[L_\mu,R_\mu,B^0_\mu\right]} = \int \mathrm{d}\left[\Sigma\right] \mathrm{e}^{\mathrm{i}S_{\mathrm{SB}}\left[\Sigma,{}^tL_\mu,{}^tR_\mu,{}^tB^0_\mu\right]+\ln\mathrm{Det}\,\hat{\mathcal{D}}_{\mathrm{c}}\left[{}^t\hat{L}_\mu,{}^t\hat{R}_\mu,{}^tB^0_\mu\right]+\mathcal{O}(p^4)} \tag{3.30}$$

$$= \int \mathrm{d}\left[\Sigma\right] \mathrm{e}^{D(\alpha,\beta,\alpha^0)}\mathrm{e}^{\mathrm{i}S_{\mathrm{SB}}\left[\Sigma,L_\mu,R_\mu,B^0_\mu\right]+\ln\mathrm{Det}\,\hat{\mathcal{D}}_{\mathrm{c}}\left[\hat{L}_\mu,\hat{R}_\mu,B^0_\mu\right]+\mathcal{O}(p^4)}\,, \tag{3.31}$$

where we have renamed the integration variables and then used invariance of the Goldstone boson measure. This is in fact what allowed us to move the differential operator $D(\alpha,\beta,\alpha^0)$ inside the functional integral, even though it also acts on the pion fields. Some definitions are in order: the action for the Goldstone bosons $S_{\mathrm{SB}}[\Sigma, L_\mu, R_\mu, B^0_\mu]$ is split into two parts

$$S_{\mathrm{SB}}\left[\Sigma, L_\mu, R_\mu, B^0_\mu\right] = S_{\mathrm{inv}}\left[\Sigma, L_\mu, R_\mu, B^0_\mu\right] + \hat{S}_{\mathrm{WZ}}\left[\Sigma, L_\mu, R_\mu, B^0_\mu\right], \tag{3.32}$$

where S_{inv} is invariant under a generic t transformation

$$D(\alpha,\beta,\alpha^0) S_{\mathrm{inv}}\left[\Sigma, L_\mu, R_\mu, B^0_\mu\right] = 0, \tag{3.33}$$

while $\hat{S}_{\mathrm{WZ}}$ is not. We still have to determine what its variation should be, depending on δ_L, δ_R and $B-L$, in order to satisfy the requirement of anomaly-matching, if this requirement can be fulfilled at all. The ambiguity in the splitting (3.32) is removed provided we adopt a boundary condition such as

$$\hat{S}_{\mathrm{WZ}}\left[\mathbb{1}, L_\mu, R_\mu, B^0_\mu\right] = 0, \tag{3.34}$$

to pin down the expression for $\hat{S}_{\mathrm{WZ}}$, which we are looking for: the anomaly-matching will be satisfied if we can solve for $\hat{S}_{\mathrm{WZ}}$. In connection with this problem, we point out that a related analysis has been performed in geometrical language in [45].

Retaining terms in (3.30) that are linear in the parameters of the transformation α, β, α^0 and using the low-energy representation for $\Gamma[L_\mu, R_\mu, B^0_\mu]$, we arrive at

$$0 = \int \mathrm{d}\left[\Sigma\right] \mathrm{e}^{\mathrm{i}S_{\mathrm{SB}}\left[\Sigma,L_\mu,R_\mu,B^0_\mu\right]+\ln\mathrm{Det}\,\hat{\mathcal{D}}_{\mathrm{c}}\left[\hat{L}_\mu,\hat{R}_\mu,B^0_\mu\right]+\mathcal{O}(p^4)} \{X[\Sigma, L_\mu, R_\mu, B^0_\mu] + \mathcal{O}(p^6)\}\,, \tag{3.35}$$

where X is defined through

$$X[\Sigma, L_\mu, R_\mu, B^0_\mu] = D(\alpha,\beta,\alpha^0)\left\{\hat{S}_{\mathrm{WZ}}\left[\Sigma, L_\mu, R_\mu, B^0_\mu\right] - \mathrm{i}\ln\mathrm{Det}\,\hat{\mathcal{D}}_{\mathrm{c}}\left[\hat{L}_\mu, \hat{R}_\mu, B^0_\mu\right]\right\} - \\ -D(\alpha,\beta,\alpha^0)\Gamma\left[L_\mu, R_\mu, B^0_\mu\right]. \tag{3.36}$$

Expanding (3.35) in powers of sources, we deduce by iteration that X vanishes when considered as a power series in the sources. Therefore, for our purposes

$$X[\Sigma, L_\mu, R_\mu, B^0_\mu] = 0\,, \tag{3.37}$$

and, from the definition for $X[\Sigma, L_\mu, R_\mu, B^0_\mu]$ (3.36), we deduce what the variation of $\hat{S}_{\mathrm{WZ}}$ must be in order to satisfy the anomaly-matching condition. Indeed, injecting the expression for the total variation from the techni-theory (3.8) and the variation of the fermion determinant (3.23) into this definition for $X[\Sigma, L_\mu, R_\mu, B^0_\mu]$, we can express the variation of $\hat{S}_{\mathrm{WZ}}[\Sigma, L_\mu, R_\mu, B^0_\mu]$ under a generic transformation t as follows

$$\begin{aligned} D(\alpha,\beta,\alpha^0)\hat{S}_{\mathrm{WZ}}\left[\Sigma, L_\mu, R_\mu, B^0_\mu\right] &= -k_{\mathrm{TT}}\int \mathrm{d}x\,\langle \beta\Omega\left[L_\mu, R_\mu, B^0_\mu\right]\rangle - \\ &\quad -(B-L)\int \mathrm{d}x\left\langle \beta\Omega\left[\hat{L}_\mu, \hat{R}_\mu, B^0_\mu\right]\right\rangle. \end{aligned} \tag{3.38}$$

One may rewrite this using the separation (3.24) between standard variation $\Omega[L_\mu, R_\mu, B^0_\mu]$ and non-standard variation $\Omega_{\delta_L,\delta_R}[L_\mu, R_\mu, B^0_\mu]$. We have already discussed the Wess-Zumino integrability conditions for both pieces in (3.38) in sections 3.1 and 3.2. The outcome was that it is possible to build a lagrangian reproducing the variation represented by each of these two terms separately, whatever the values of δ_L, δ_R. Here we only have to build a linear combination of the corresponding Wess-Zumino lagrangians: the standard one and the non-standard one. Indeed, a particular solution to this non-standard Wess-Zumino problem is given by

$$\hat{S}_{\mathrm{WZ}}[\Sigma, L_\mu, R_\mu, B^0_\mu] = (k_{\mathrm{TT}} + B - L)S_{\mathrm{WZ}}[\Sigma, L_\mu, R_\mu, B^0_\mu] + (B-L)S_{\delta_L,\delta_R}[\Sigma, L_\mu, R_\mu, B^0_\mu]\,. \tag{3.39}$$

The boundary conditions for both terms in the last equation are given in (3.15) and (3.28): they imply that our construction automatically verifies the initial condition (3.34) we have chosen.

Thus, equation (3.39) indeed gives us an action $\hat{S}_{\mathrm{WZ}}$ satisfying (3.38). This shows that the anomaly-matching condition can be solved whatever the value of δ_L, δ_R and $B-L$, and that there are therefore no restrictions on these constants. In addition, we have obtained the expression of the non-standard Wess-Zumino lagrangian to be added to the bosonic part of the effective lagrangian in order for the total variation of the generating functional to be recovered. Our result obviously accounts for the case without composite fermions as well, as can be found by setting $B-L=0$ in equation (3.39). The low-energy representation of the generating functional Γ reads, with the various pieces introduced above

$$\mathrm{e}^{\mathrm{i}\Gamma\left[L_\mu,R_\mu,B^0_\mu\right]} = \int \mathrm{d}\left[\Sigma\right]\mathrm{e}^{\mathrm{i}S_{\mathrm{inv}}\left[\Sigma,L_\mu,R_\mu,B^0_\mu\right]+\mathrm{i}\hat{S}_{\mathrm{WZ}}\left[\Sigma,L_\mu,R_\mu,B^0_\mu\right]+\ln\mathrm{Det}\,\mathcal{D}_{\mathrm{c}}\left[\hat{L}_\mu,\hat{R}_\mu,B^0_\mu\right]+\mathcal{O}(p^4)}\,. \tag{3.40}$$

Coming back to the assumed transformation properties for the composite fermions, we may also consider the case where the left and right-handed fermions have transformation properties others than those given by (2.11) and (2.12). Except for an interchange between the left and right-handed fermions, however, other choices lead to a Dirac operator whose determinant is invariant. Indeed, in these cases, the left and right-handed fermions do not transform independently under a transformation t, and therefore there can be no anomalous variation of the action under such a transformation. This then entails that such fermion doublets do not participate in the anomaly-matching.

4. Anomaly-matching in a Higgs-less effective theory

If we were to study the anomaly-matching in a theory where the Goldstone modes remained in the spectrum, the results of the previous section would be sufficient: there we performed the anomaly-matching, even in the case where there are composite fermions in the spectrum. However, this is not the end of the story, as we are interested in the situation where the (dynamical) Higgs mechanism occurs, resulting in the removal of all Goldstone modes from the spectrum.

At this stage, the gauge fields $G_{0\mu}, G_{1\mu}$ introduced in section 2.2 and coupled to the other fields via spurions come into play. Indeed, the spurions introduce the proper identification between the gauge fields and the chiral sources L_μ, R_μ, B^0_μ, which appeared directly in the discussion of anomalies. The situation is analogous to that of χPT, where the global anomalies derived from QCD have to be reproduced by the LEET. This is the anomaly-matching discussed in the previous section, which is implemented by adding a Wess-Zumino term in the LEET, with particular care paid to possible composite fermions. However, we are interested in theories of EWSB where all Goldstone modes can be gauged away and the electroweak gauge bosons get their masses. We will require that the techni-theory be such that this indeed happens: the lagrangian can then be rewritten in terms of variables not involving Goldstone fields anymore. Consequently, we have to study field redefinitions related to gauge transformations in order to determine whether the Σ field can indeed be eliminated. Anomalies will play an important part in the discussion at $\mathcal{O}(p^4)$ and higher orders. As it turns out, the conclusion will be that such a redefinition is impossible unless anomalies for the low-energy symmetries of the techni-theory vanish, that is, $k_{\mathrm{TT}} = 0$. This is a general condition on the techni-theory itself.

This result is the same whether or not the techni-theory produces composite fermions. Indeed, neither $B - L$ nor the non-standard couplings δ_L, δ_R will ever appear in the manipulations to be performed: using the result of the anomaly-matching of section 3, the question is phrased in terms of the content of the techni-theory itself, instead of the low-energy variables. Concerning elementary fermions, the consequence will be that the trace of $B - L$ over these elementary fermions has to vanish.

4.1 Anomaly obstruction: the composite sector

We focus on the definition of the unitary gauge using field transformations, in order to check whether the π^a fields can be eliminated from the action. These redefinitions of course rely on the gauge invariance of the action and we ask what is their effect in presence of anomalies, that is, if the constant k_{TT} appearing as a multiplicative factor in front of the anomalous variation of the generating functional for the techni-theory (3.8) is non-zero. We will again study this question at the first order where it appears, that is $\mathcal{O}(p^4\epsilon^0\zeta^0)$. Taking into account possible composite fermions, but not elementary fermions in this section, we can in fact once more work at the stage before the gauge fields $G_{0\mu}, G_{1\mu}$ are introduced, as we will see.

We work at the level where the fermions are integrated out to yield a determinant, allowing us to focus at first on the bosons. We will briefly come back to the field redefinitions for the composite fermions in section 5. For the case at hand, we will consider the action $S\left[\Sigma, L_\mu, R_\mu, B_\mu^0\right]$

$$
\begin{aligned}
S\left[\Sigma, L_\mu, R_\mu, B_\mu^0\right] = & \; S_{\mathrm{inv}}\left[\Sigma, L_\mu, R_\mu, B_\mu^0\right] + \hat{S}_{\mathrm{WZ}}\left[\Sigma, L_\mu, R_\mu, B_\mu^0\right] - \\
& -\mathrm{i} \ln \mathrm{Det}\, \hat{\mathcal{D}}_{\mathrm{c}}\left[\hat{L}_\mu, \hat{R}_\mu, B_\mu^0\right], \qquad (4.1)
\end{aligned}
$$

which occurs in the low-energy expression for $\Gamma\left[L_\mu, R_\mu, B_\mu^0\right]$ (3.40). Integration over the gauge fields will be performed at a later stage. However, one has to keep in mind that the three vector sources R_μ^a will be identified with the dynamical fields $G_{1\mu}^a$, hence a field redefinition involving these sources can absorb the three Goldstone bosons, while the three sources L_μ^a only contain one dynamical field $L_\mu^3 = g_0 G_{0\mu}^3$ and cannot be used for the same purpose. The assumed disappearance of the Goldstone modes from the spectrum means that the action can be rewritten in terms of only seven vector sources, but from the reasoning above, we see that we need only redefine the R_μ sources. We will explicitly use the following definition in order to obtain a new set of vector variables invariant under the $\mathrm{SU}(2)_R$ symmetry

$$
g_1 W_\mu = \mathrm{i}\Sigma D_\mu \Sigma^\dagger = \Sigma R_\mu \Sigma^\dagger + \mathrm{i}\Sigma\partial_\mu\Sigma^\dagger - L_\mu\,, \qquad (4.2)
$$

and perform the change of variables

$$
\left\{\Sigma, L_\mu, R_\mu, B_\mu^0\right\} \longrightarrow \left\{\Sigma, L_\mu, g_1 W_\mu, B_\mu^0\right\}. \qquad (4.3)
$$

The possibility to define the action in terms of variables invariant under the gauge symmetry is a consequence of a well-known aspect of the Higgs mechanism —the screening of the gauge charges. The question is then whether, using the general writing

$$
S\left[\Sigma, L_\mu, R_\mu, B_\mu^0\right] = f\left[L_\mu, g_1 W_\mu, B_\mu^0\right] + h\left[\Sigma, L_\mu, g_1 W_\mu, B_\mu^0\right], \qquad (4.4)
$$

where the h functional a priori depends on Σ, we find or not

$$
h\left[\Sigma, L_\mu, g_1 W_\mu, B_\mu^0\right] = 0\,. \qquad (4.5)
$$

Again, to avoid ambiguities in the separation (4.4), we impose the condition

$$
h\left[\mathbb{1}, L_\mu, g_1 W_\mu, B_\mu^0\right] = 0\,. \qquad (4.6)
$$

We first make use of the fact that (4.2) is formally a gauge transformation. Indeed we can write

$$
g_1 W_\mu =^{\omega} R_\mu - L_\mu\,, \qquad (4.7)
$$

with the transformation parameter chosen to depend on the Goldstone fields in the following manner

$$\omega = \left(L = \mathbb{1}, R = \Sigma, \alpha^0 = 0\right). \tag{4.8}$$

This would indeed be sufficient if only the invariant part in the action (4.1) were present, as we have

$$S_{\text{inv}}\left[\Sigma, L_\mu, R_\mu, B^0_\mu\right] = S_{\text{inv}}\left[\mathbb{1}, g_1 W_\mu + L_\mu, L_\mu, B^0_\mu\right]. \tag{4.9}$$

Note that we really are only performing field redefinitions and not fixing a gauge [46, 47], therefore there will be no conflict with the fact that we have to choose a particular gauge to solve later the constraints on the spurions, as done in section 2.2: the gauge transformations are merely invoked to guide our reasoning.

We can now study the consequences on the total action S. In fact, we first consider the case of infinitesimal π^a fields and use the result of the anomaly-matching for an infinitesimal transformation

$$D\left(\alpha, \beta, \alpha^0\right) S\left[\Sigma, L_\mu, R_\mu, B^0_\mu\right] = -k_{\text{TT}} \int \mathrm{d}x \left\langle \beta\Omega\left[L_\mu, R_\mu, B^0_\mu\right]\right\rangle. \tag{4.10}$$

We next inject in this relation the infinitesimal version of the chiral transformation (4.8) used above in order to eliminate the Goldstone fields from the even intrinsic-parity terms. This transformation assumes the form

$$\omega = \left(L = \mathbb{1}, R = 1 + \mathrm{i}\frac{\pi}{f} + \mathcal{O}\left(\pi^2\right), \alpha^0 = 0\right), \tag{4.11}$$

where the Goldstone fields π were defined in (3.5). Injecting (4.11) into (4.10) and reshuffling the terms, the result (4.10) can be recast in a form more useful for our purposes

$$S\left[\Sigma, L_\mu, R_\mu, B^0_\mu\right] = S[\mathbb{1}, L_\mu, g_1 W_\mu + L_\mu, B^0_\mu] + \frac{k_{\text{TT}}}{2f} \int \mathrm{d}x \langle \pi\Omega[L_\mu, g_1 W_\mu + L_\mu, B^0_\mu]\rangle + \mathcal{O}(\pi^2). \tag{4.12}$$

We can identify the part independent on the Goldstone boson fields

$$f\left[L_\mu, g_1 W_\mu, B^0_\mu\right] = S\left[\mathbb{1}, L_\mu, g_1 W_\mu + L_\mu, B^0_\mu\right], \tag{4.13}$$

whereupon we see that the power series of h in powers of π^a begins with the linear term

$$h\left[\Sigma, L_\mu, g_1 W_\mu, B^0_\mu\right] = \frac{k_{\text{TT}}}{2f} \int \mathrm{d}x \left\langle \pi\Omega\left[L_\mu, g_1 W_\mu + L_\mu, B^0_\mu\right]\right\rangle + \mathcal{O}\left(\pi^2\right). \tag{4.14}$$

The only possibility for this functional to be independent of the Goldstone modes is then to have

$$k_{\text{TT}} = 0, \tag{4.15}$$

that is, if the symmetries of the techni-theory that are operative on the low-energy degrees of freedom are anomaly-free. In that case only can we get rid of the Goldstone fields via field redefinitions. We now show that this remains true for finite values of the Goldstone fields.

If condition (4.15) is met by the techni-theory, we can show that the field redefinition (4.2) does eliminate the Goldstone boson field in the finite case as well: we then have

$$\begin{aligned} S\left[\mathbb{1}, L_\mu, g_1 W_\mu + L_\mu, B^0_\mu\right] &= \mathrm{e}^{D(\alpha,\beta,0)} S\left[\Sigma, L_\mu, R_\mu, B^0_\mu\right]\Big|_{(\alpha,\beta)=\left(\frac{\pi}{2f},\frac{\pi}{2f}\right)} \\ &= S\left[\Sigma, L_\mu, R_\mu, B^0_\mu\right], \end{aligned} \tag{4.16}$$

where, in the last step, we have used the absence of anomalous variation of $S[\Sigma, L_\mu, R_\mu, B^0_\mu]$ in (4.10) when $k_{\rm TT} = 0$, as deduced in (4.15). The functional $h[\Sigma, L_\mu, g_1 W_\mu, B^0_\mu]$, defined in (4.4), is then found to vanish, concluding our proof.

To summarize the main result of the present section, we recall that the condition that the Goldstone modes do not appear in the spectrum requires the absence of anomalies for the low-energy symmetries of the techni-theory: $k_{\rm TT}$ has to vanish. If there are light composite fermions, the LEET must involve a Wess-Zumino-like term in order to compensate for the anomalous variation of the fermion determinant. We have shown that such a Wess-Zumino term can be constructed whatever the value of δ_L, δ_R and $B-L$. On the other hand, if there are no light composite fermions, then there will be no Wess-Zumino term, since $k_{\rm TT}$ has to vanish.

We have concluded that, for those Higgs-less models to which our effective description applies, the underlying theory has to be anomaly-free with respect to the global group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$, in order for the GBs to disappear from the spectrum. Considering applications to technicolor models, one must keep in mind that the larger $S_{\rm natural}$ symmetry has never been introduced in the past. On the other hand, technicolor models are usually constructed to be anomaly-free under the electroweak gauge group $\mathrm{SU}(2) \times \mathrm{U}(1)_Y$ [48], which is the corresponding requirement. In order for anomalies to vanish, one adds new particles, for instance strongly-interacting technileptons to balance the $B-L$ charges of the techniquarks. The global symmetry group is then larger than that of QCD with two massless flavors (i.e. than $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$). One expects this modification to be reflected in the spectrum of (massive) bound states, and therefore the spectral functions should *a priori* behave differently. Therefore, the estimates for the value of the S parameter (alternatively L_{10}), derived from QCD are not theoretically justified in the cases of interest for Higgs-less EWSB, since they are directly linked with the vector and axial spectral functions [10].

We stress again that, using the result from the anomaly-matching derived previously, we did not have to deal with the couplings of possible composite fermions: these simply did not show up in the reasoning. As far as the Higgs mechanism is concerned, composite fermions may just as well be absent: the derivation would be identical.

Actually, as far as anomalies are concerned, the main difference between composite fermions (if they are present) and elementary ones are concerns the trace of the quantum number $B-L$. Whereas for elementary fermions, $\sum(B-L)$ has to vanish (see next section), for composite fermions this is not necessary: composite fermions do not necessarily constitute a full generation as the SM fermions do. There is in fact an additional distinction stemming from the mass term (2.16): compared to elementary fermions, composite fermions are heavier (see [25]) but have a relatively smaller mass-splitting within the doublet. Obviously, these are theoretical distinctions, and composite fermions might be mistaken for elementary ones in experiments, especially if $\delta_L \simeq \delta_R \simeq 0$.

4.2 Elementary fermions in the unitary gauge

In this section, we consider elementary fermions, instead of the composite fermions we have been dealing with up to now. For these fermions, there is no anomaly-matching to be performed since the fundamental variables are assumed to correspond directly to the physical degrees of freedom at low-energy, due to the perturbative nature of their couplings. We thus focus on the question of anomalies as an obstruction to the elimination of the Goldstone modes from the action: as we shall see, the results are independent of our previous discussion concerning composite fermions. Elementary fermions have been studied in an exactly identical setting in [25], and we only discuss additional aspects related to anomalies. We will find that the trace of $B-L$ over the elementary fermions has to vanish for the action to be independent of the Goldstone modes.

We define the Dirac operator $\mathcal{D}_{\mathrm{e}}$

$$\mathcal{D}_{\mathrm{e}}\chi^{\mathrm{e}} = \mathrm{i}\gamma^{\mu} D_{\mu}\chi^{\mathrm{e}}\,, \tag{4.17}$$

such that the $\mathcal{O}(p^2)$ lagrangian for elementary fermions is given by

$$\mathcal{L}_{\mathrm{elem\,.\,ferm\,.}} = \overline{\chi^{\mathrm{e}}}\mathcal{D}_{\mathrm{e}}\chi^{\mathrm{e}}\,. \tag{4.18}$$

The transformation properties of $\mathcal{D}_{\mathrm{e}}$ under the symmetries $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ of the techni-theory are trivial given that the elementary fermions on which it is to be applied are external to this theory, and we need only consider $\mathrm{SU}(2)_{G_0} \times \mathrm{SU}(2)_{G_1} \times \mathrm{U}(1)_{B-L}$ transformations. Under such a transformation τ parameterized

$$\tau = \left(G_0 = \mathrm{e}^{\mathrm{i}(\theta-\varphi)}, G_1 = \mathrm{e}^{\mathrm{i}(\theta+\varphi)}, \alpha^0\right), \tag{4.19}$$

the variation of the determinant is given by the following expression, where we restrict to the first non-zero term, that is $\mathcal{O}(p^4\epsilon^0)$

$$-\mathrm{i}D(\theta,\varphi,\alpha^0)\ln \mathrm{Det}\,\mathcal{D}_{\mathrm{e}}[g_0G_{0\mu}, g_1G_{1\mu}, B^0_{\mu}] = (B-L)\int \mathrm{d}x\langle\varphi\Omega[g_0G_{0\mu}, g_1G_{1\mu}, B^0_{\mu}]\rangle\,. \tag{4.20}$$

The Bardeen functional Ω occurring here is the same as appeared previously in (3.10) albeit with different functional arguments. We may explicitly display its expression as

$$\Omega\left[g_0G_{0\mu}, g_1G_{1\mu}, B^0_{\mu}\right] = \frac{1}{16\pi^2}\varepsilon^{\mu\nu\rho\sigma}B^0_{\mu\nu}\left(g_0\partial_\rho G_{0\sigma} + g_1\partial_\rho G_{1\sigma} - \mathrm{i}g_0g_1\left[G_{0\rho}, G_{1\sigma}\right]\right). \tag{4.21}$$

We now turn to the field redefinitions required to obtain the unitary gauge variables for the bosonic sector. Since the elementary fermions do not transform under $\mathrm{SU}(2)_R$ but rather under $\mathrm{SU}(2)_{G_1}$, we have to redefine the $G_{1\mu}$ fields rather than the R_μ sources as we did in (4.2) in order to absorb the three Goldstone bosons. We would like to perform the field redefinitions at the level of the full covariant lagrangian, that is, regardless of any gauge choice. Notice that the real spurion X, connecting R_μ and $g_1 G_{1\mu}$ can be decomposed as

$$X = \xi(x) U(x) \,, \tag{4.22}$$

where $\xi(x)$ is a real function and $U \in \mathrm{SU}(2)$. The appropriate redefinition of the fields $G^a_{1\mu}$ that absorbs the matrix Σ into the three vector fields $\mathcal{W}^a_\mu$ is then

$$\begin{aligned} g_1 \mathcal{W}_\mu &= \mathrm{i}\Sigma U D_\mu \left(U^\dagger \Sigma^\dagger\right) \\ &= \mathrm{i}\Sigma U \partial_\mu \left(U^\dagger \Sigma^\dagger\right) + g_1 \Sigma U G_{1\mu} U^\dagger \Sigma^\dagger - L_\mu \,, \end{aligned} \tag{4.23}$$

instead of the definition previously given in (4.2). On the space of solutions of the constraints $D_\mu X = 0$ the two definitions are actually identical. Indeed, the constraint of covariant constancy applied on the spurion X, results in ξ being a constant and into

$$D_\mu U \equiv \partial_\mu U - \mathrm{i} R_\mu U + \mathrm{i} g_1 U G_{1\mu} = 0 \,, \tag{4.24}$$

showing that

$$\mathcal{W}_\mu = W_\mu \,. \tag{4.25}$$

This result is true as it stands, due to the constraints, and without any gauge choice. We notice that the field redefinition (4.23) is again formally a gauge transformation, but this time it is a particular case of a τ transformation as introduced in (4.19) — as opposed to a t transformation (3.1) — with parameters depending on the Goldstone and spurion fields

$$\lambda = \left(G_0 = \mathbb{1}, G_1 = \Sigma U, \alpha^0 = 0\right) , \tag{4.26}$$

so that we can rewrite (4.23) as

$$g_1 \mathcal{W}_\mu = g_1 {}^\lambda G_{1\mu} - L_\mu \,. \tag{4.27}$$

To study the consequences of the field redefinition (4.23), we use the outcome of the integration

$$\int \mathrm{d}\left[\chi^{\mathrm{e}}\right] \mathrm{e}^{\mathrm{i}\int \mathrm{d}x \overline{\chi^{\mathrm{e}}} \mathcal{D}_{\mathrm{e}}\left[g_0 G_{0\mu}, g_1 G_{1\mu}, B^0_\mu\right] \chi^{\mathrm{e}}} = \mathrm{e}^{\ln \mathrm{Det}\, \mathcal{D}_{\mathrm{e}}\left[g_0 G_{0\mu}, g_1 G_{1\mu}, B^0_\mu\right]} \,, \tag{4.28}$$

and proceed as in section 4.1. We start with the case of an infinitesimal π field and infinitesimal κ of the same order defined as

$$U = \mathrm{e}^{\mathrm{i}\frac{\kappa}{f}} \,. \tag{4.29}$$

Application of an infinitesimal transformation then yields

$$-\mathrm{i} D\left(\theta = \frac{\pi + \kappa}{2f}, \varphi = \frac{\pi + \kappa}{2f}, \alpha^0 = 0\right) \ln \mathrm{Det}\, \mathcal{D}_{\mathrm{e}} \left[g_0 G_{0\mu}, g_1 G_{1\mu}, B^0_\mu\right] =$$

$$= \frac{B-L}{2f} \int \mathrm{d}x \left\langle (\pi+\kappa)\, \Omega \left[g_0 G_{0\mu}, g_1 \mathcal{W}_\mu + L_\mu, B^0_\mu \right] \right\rangle + \mathcal{O}(\pi^2, \pi\kappa, \kappa^2)\,. \quad (4.30)$$

This shows that it is impossible to absorb the Goldstone boson fields in the action unless the trace of $B-L$ over elementary fermions vanishes. To deduce this, we have used the fact that there can be no cancellation between (4.30) and (4.12): the anomalous variation of the determinant for elementary fermions must vanish by itself in order to get rid of the Goldstone modes. Thus we must impose for consistency

$$\sum_{\text{elementary fermions}} (B-L) = 0\,. \quad (4.31)$$

One then checks that the field redefinitions do also work for finite transformations. Usually, the condition (4.31) derives from the requirement of renormalizability of the theory [49, 46, 50, 51], which is not of an obvious relevance within the present framework of LEET. Nevertheless, this relation reappears here as a necessary condition for the Higgs-less symmetry breaking mechanism to occur.

Finally, in order to write the action in unitary gauge at the level of the functional integral for fermions, we may in addition to the field redefinitions in the bosonic sector, introduce the following redefinition for fermions

$$\psi^{\mathrm{e}}_L = {}^{\lambda}\chi^{\mathrm{e}}_L = \Sigma U \chi^{\mathrm{e}}_L\,, \quad (4.32)$$
$$\psi^{\mathrm{e}}_R = {}^{\lambda}\chi^{\mathrm{e}}_R = \chi^{\mathrm{e}}_R\,. \quad (4.33)$$

This is in fact the definition implied by the transformation λ (4.26). The new ψ^{e} variables are invariant under the $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_{G_1}$ symmetry as are the W_μ fields: this is indeed the purpose of these redefinitions

$$\psi^{\mathrm{e}}_L \longmapsto {}^{t}\psi^{\mathrm{e}}_L = L \mathrm{e}^{-\mathrm{i}\frac{B-L}{2}\alpha^0} \psi^{\mathrm{e}}_L\,, \quad (4.34)$$
$$\psi^{\mathrm{e}}_R \longmapsto {}^{t}\psi^{\mathrm{e}}_R = G_0 \mathrm{e}^{-\mathrm{i}\frac{B-L}{2}\alpha^0} \psi^{\mathrm{e}}_R\,. \quad (4.35)$$

Since the lagrangian for elementary fermions (2.27), as well as the fermionic integration measure are both invariant under the transformation λ, we have

$$\int \mathrm{d}\left[\chi^{\mathrm{e}}\right] \mathrm{e}^{\mathrm{i}\overline{\chi^{\mathrm{e}}} \mathcal{D}_{\mathrm{e}}\left[g_0 G_{0\mu}, g_1 G_{1\mu}, B^0_\mu\right] \chi^{\mathrm{e}}} = \int \mathrm{d}\left[\psi^{\mathrm{e}}\right] \mathrm{e}^{\mathrm{i}\overline{\psi^{\mathrm{e}}} \mathcal{D}_{\mathrm{e}}\left[g_0 G_{0\mu}, g_1 \mathcal{W}_\mu + L_\mu, B^0_\mu\right] \psi^{\mathrm{e}}}\,. \quad (4.36)$$

The conclusion is then that the lagrangian in the unitary gauge is directly obtained via the replacement

$$\left\{\chi^{\mathrm{e}}, \Sigma, G_{1\mu}, B^0_\mu\right\} \longrightarrow \left\{\psi^{\mathrm{e}}, \mathbb{1}, \mathcal{W}_\mu + \frac{1}{g_1} L_\mu, B^0_\mu\right\}. \quad (4.37)$$

5. Triangles in the unitary gauge

We now wish to introduce the corresponding redefinitions for composite fermions in the unitary gauge. They should be such that the composite fermion fields are invariant under $\mathrm{SU}(2)_R$. Therefore, we set, using the ω transformation from (4.8)

$$\psi^{\mathrm{c}}_L = {}^{\omega}\chi^{\mathrm{c}}_L = \Sigma \chi^{\mathrm{c}}_L\,, \quad (5.1)$$

$$\psi_R^{\rm c} = {}^{\omega}\chi_R^{\rm c} = \chi_R^{\rm c}\,, \tag{5.2}$$

which indeed yields

$$\psi_L^{\rm c} \longmapsto {}^{t}\psi_L^{\rm c} = L\mathrm{e}^{-\mathrm{i}\frac{B-L}{2}\alpha^0}\psi_L^{\rm c}\,, \tag{5.3}$$
$$\psi_R^{\rm c} \longmapsto {}^{t}\psi_R^{\rm c} = L\mathrm{e}^{-\mathrm{i}\frac{B-L}{2}\alpha^0}\psi_R^{\rm c}\,, \tag{5.4}$$

as desired.

Of course, the determinant of the Dirac operator $\mathcal{D}_{\rm c}$ defined as $\mathcal{D}_{\rm c}\left[\Sigma, L_\mu, R_\mu, B_\mu^0\right] = \hat{\mathcal{D}}_{\rm c}\left[\hat{L}_\mu, \hat{R}_\mu, B_\mu^0\right]$ is not invariant under the ω transformation. However, due to the absence of anomalies for the whole techni-theory ($k_{\rm TT} = 0$), this variation is compensated by that of the Wess-Zumino term. Hence we can rewrite (4.16) in the form

$$\mathrm{e}^{\mathrm{i}S_{\rm inv}\left[\Sigma,L_\mu,R_\mu,B_\mu^0\right]+\mathrm{i}\hat{S}_{\rm WZ}\left[\Sigma,L_\mu,R_\mu,B_\mu^0\right]}\int \mathrm{d}\left[\chi^{\rm c}\right]\mathrm{e}^{\mathrm{i}\int \mathrm{d}x\overline{\chi^{\rm c}}\mathcal{D}_{\rm c}\left[\Sigma,L_\mu,R_\mu,B_\mu^0\right]\chi^{\rm c}} =$$
$$= \mathrm{e}^{\mathrm{i}S_{\rm SB}\left[\mathbb{1},L_\mu,g_1W_\mu+L_\mu,B_\mu^0\right]}\int \mathrm{d}\left[\psi^{\rm c}\right]\mathrm{e}^{\mathrm{i}\int \mathrm{d}x\overline{\psi^{\rm c}}\mathcal{D}_{\rm c}\left[\mathbb{1},L_\mu,g_1W_\mu+L_\mu,B_\mu^0\right]\psi^{\rm c}}\,, \tag{5.5}$$

where we have used the boundary condition (3.34) for $\hat{S}_{\rm WZ}$. The outcome is again that we merely have to perform the following replacement in the lagrangian

$$\left\{\chi^{\rm c}, \Sigma, L_\mu, R_\mu, B_\mu^0\right\} \longmapsto \left\{\psi^{\rm c}, \mathbb{1}, L_\mu, g_1W_\mu + L_\mu, B_\mu^0\right\}\,, \tag{5.6}$$

in order to obtain the lagrangian in unitary gauge.

It is in fact quite natural to work in the unitary gauge for such effective theories: this has the additional advantage that the Wess-Zumino-like term vanishes in this gauge, due to the boundary condition (3.34). We therefore use this gauge for the following discussion of anomalous triangular diagrams: we can write

$$\int \mathrm{d}\left[\Sigma\right]\mathrm{e}^{\mathrm{i}S\left[\Sigma,L_\mu,R_\mu,B_\mu^0\right]} = \int \mathrm{d}\left[\psi^{\rm c}\right]\mathrm{e}^{\mathrm{i}\int \mathrm{d}x\mathcal{L}_{\rm SB}\left[\mathbb{1},L_\mu,g_1W_\mu+L_\mu,B_\mu^0\right]+\overline{\psi^{\rm c}}\mathcal{D}_{\rm c}\left[\mathbb{1},L_\mu,g_1W_\mu+L_\mu,B_\mu^0\right]\psi^{\rm c}}\,, \tag{5.7}$$

where the integration over Σ has been performed, yielding a constant factor since the action does not depend on these fields anymore.

To determine the anomalous contribution of fermion triangles to the three-point functions of the techni-currents, we may proceed as follows: first one uses the expression for the techni-currents J_L^μ in terms of the low-energy variables. These are obtained by considering the full lagrangian in a generic gauge, taking the functional derivative with respect to the sources L_μ,[6] and then injecting the solution to the constraints in the standard gauge to make the field content explicit. When this is written in terms of unitary gauge variables, we obtain[7]

$$J_L^{3\nu} = -\sqrt{1+\gamma^2}\frac{M_W^2}{g_1}Z^\nu + \delta_L\overline{\psi_L^{\rm c}}\gamma^\nu\frac{\tau^3}{2}\psi_L^{\rm c} + (1-\delta_R)\,\overline{\psi_R^{\rm c}}\gamma^\nu\frac{\tau^3}{2}\psi_R^{\rm c}\,, \tag{5.8}$$

[6] While the currents J_R^μ are perfectly sensible in the techni-theory, their expression in terms of low-energy variables would be of little use in the present framework since the corresponding sources R^μ are identified with the SU(2) gauge fields $g_1G_1^\mu$.

[7] The necessary diagonalization for vector fields was described in [25].

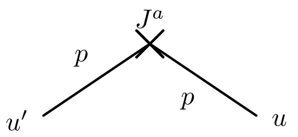

Figure 6: Direct contribution to the matrix element.

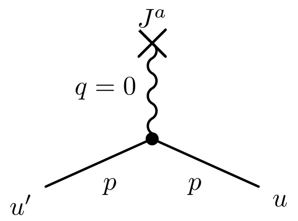

Figure 7: Indirect contribution to the matrix element, involving the emission of a vector boson.

$$J_L^{\pm\nu} = -\frac{M_W^2}{g_1} W^{\nu\pm} + \frac{1}{\sqrt{2}} \delta_L \overline{\psi_L^{\mathrm{c}}} \gamma^\nu \tau^\pm \psi_L^{\mathrm{c}} + \frac{1}{\sqrt{2}} \left(1 - \delta_R\right) \overline{\psi_R^{\mathrm{c}}} \gamma^\nu \tau^\pm \psi_R^{\mathrm{c}} , \tag{5.9}$$

where

$$M_W^2 = g_1^2 \frac{f^2}{4} , \tag{5.10}$$

$$\gamma = \frac{g_0}{g_1} . \tag{5.11}$$

We can then express, following [27], the matrix element of these currents between two asymptotic fermion states described by spinors u' and u satisfying the massless Dirac equation, with identical momentum p

$$\left\langle u', i, p \left| J_L^{a\mu}(0) \right| u, j, p \right\rangle = \overline{u'} \gamma^\mu \mathcal{J}_{ij}^a u , \tag{5.12}$$

where $\mathcal{J}^a$ is a two by two matrix representing the charges associated to the Noether currents $J_L^{a\mu}$ (the indices i, j distinguish between the upper and lower component of the doublet).

To calculate this, we must take into account two contributions: one coming from a direct contraction of the external spinors with the last two terms in (5.8) or in (5.9), as depicted in figure 6, and one where a vector propagator is involved, as in figure 7. However in this last case, the propagator is taken at $k = 0$, and this results in a local term. The matrix element of (5.12) is then given by the sum of the two terms. In the second graph (figure 7), the longitudinal polarization of the vector propagator does not contribute, as can be seen by making use of the massless Dirac equation. Calculation of this last diagram requires the expression for the interactions of composite fermions with the vector fields. This is obtained by injecting the solution to the constraints in the standard gauge into the lagrangian for composite fermions (2.13), rewriting in the unitary gauge and then diagonalizing the vector fields, to arrive at

$$\begin{aligned} \mathcal{L}_{\mathrm{comp}} &= \mathrm{i} \overline{\psi^{\mathrm{c}}} \gamma^\mu \partial_\mu \psi^{\mathrm{c}} + e \overline{\psi^{\mathrm{c}}} \gamma^\mu Q \psi^{\mathrm{c}} A_\mu - g_1 \frac{\gamma^2}{\sqrt{1+\gamma^2}} \overline{\psi^{\mathrm{c}}} \gamma^\mu Q \psi^{\mathrm{c}} Z_\mu + \\ &\quad + g_1 \left(1 - \delta_L\right) \sqrt{1+\gamma^2} \overline{\psi_L^{\mathrm{c}}} \gamma^\mu \frac{\tau^3}{2} \psi_L^{\mathrm{c}} Z_\mu + \frac{g_1}{\sqrt{2}} \left(1 - \delta_L\right) \overline{\psi_L^{\mathrm{c}}} \gamma^\mu \tau^\pm \psi_L^{\mathrm{c}} W_\mu^\pm + \\ &\quad + g_1 \delta_R \sqrt{1+\gamma^2} \overline{\psi_R^{\mathrm{c}}} \gamma^\mu \frac{\tau^3}{2} \psi_R^{\mathrm{c}} Z_\mu + \frac{g_1}{\sqrt{2}} \delta_R \overline{\psi_R^{\mathrm{c}}} \gamma^\mu \tau^\pm \psi_R^{\mathrm{c}} W_\mu^\pm , \end{aligned} \tag{5.13}$$

with

$$Q = \frac{\tau^3}{2} + \frac{B-L}{2}, \tag{5.14}$$

$$e = \frac{g_1 g_0}{\sqrt{g_1^2 + g_0^2}}. \tag{5.15}$$

Setting $\delta_L = \delta_R = 0$ in (5.13), one may check that the same tree-level couplings of fermions to vector fields as in the SM are recovered. As announced before, δ_L and δ_R respectively allow for non-universal couplings of left-handed composite fermions and for couplings of right-handed composite fermions with $W^\pm$ respectively. With this in mind, one finds that the forward matrix elements of the type (5.12) have a vector structure and do not involve the anomalous δ_L, δ_R couplings

$$\mathcal{J}^3 = \frac{1}{1+\gamma^2}\left(\frac{\tau^3}{2} - \gamma^2 \frac{B-L}{2}\right), \tag{5.16}$$

$$\mathcal{J}^\pm = \frac{\tau^\pm}{\sqrt{2}}. \tag{5.17}$$

The same is true of the matrix elements of the electromagnetic current. Thus, triangle diagrams with composite fermions running around the loop do not generate any anomalous contribution to the three-point functions of the techni-currents and electromagnetic currents. There are no other possible anomalous contributions to these Green's functions in unitary gauge since the Wess-Zumino term $\hat{S}_{\mathrm{WZ}}$ occurring in the right-hand side of (5.7) vanishes due to the boundary condition (3.34). This result agrees with that of section 4.1, which stated that the Green's functions of the techni-currents must be anomaly-free, otherwise we would not have been able to absorb the Goldstone bosons when defining the unitary gauge in the first place. The effects of δ_L and δ_R will then only be visible for $q^2 \neq 0$ in figure 7.

On the other hand, we still have to investigate the contribution of triangular diagrams to the three-point functions of the gauge bosons: anomalies were first encountered in this context in the literature. Expressing the couplings of fermions to gauge bosons in unitary gauge as given by (5.13) in terms of the following building blocks

$$j^0_\mu = \frac{1}{2}\overline{\psi^{\mathrm{c}}}\gamma_\mu \psi^{\mathrm{c}}, \tag{5.18}$$

$$j^a_\mu = \overline{\psi^{\mathrm{c}}}\gamma_\mu \frac{\tau^a}{2}\psi^{\mathrm{c}}, \tag{5.19}$$

$$j^{5,a}_\mu = \overline{\psi^{\mathrm{c}}}\gamma_\mu\gamma_5 \frac{\tau^a}{2}\psi^{\mathrm{c}}, \tag{5.20}$$

one can give the expression of the triangle contribution to an effective triple gauge boson interaction in terms of one single function $T_{\mu\nu\rho}$ (see [27]) containing the full information about the triangle

$$(2\pi)^4 \delta^{(4)}(k_1+k_2+k_3) T_{\mu\nu\rho}(k_1,k_2,k_3) = -\int \mathrm{d}x \mathrm{d}y \mathrm{d}z e^{-\mathrm{i}(k_1\cdot x + k_2 \cdot y + k_3 \cdot z)} \times$$
$$\times \langle 0 | T j^0_\mu(x) j^3_\nu(y) j^{5,3}_\rho(z) | 0 \rangle. \tag{5.21}$$

Defining

$$\mathrm{e}^{\mathrm{i}W\left[W_\mu^\pm,Z_\mu,A_\mu\right]} = \int \mathrm{d}\left[\psi^{\mathrm{c}}\right] \mathrm{e}^{\mathrm{i}\int \mathrm{d}x \mathcal{L}_{\mathrm{comp.\,ferm.}}\left[\psi^{\mathrm{c}},W_\mu^\pm,Z_\mu,A_\mu\right]}\,, \tag{5.22}$$

we get in momentum space the following trilinear terms

$$\begin{aligned} W[W_\mu^\pm,Z_\mu,A_\mu] = \int \frac{\mathrm{d}k_2}{(2\pi)^4}\frac{\mathrm{d}k_3}{(2\pi)^4} g_1^3 T_{\mu\nu\rho}(-k_2-k_3,k_2,k_3)\,\times \\ \times\Big\{ & D^{AWW}A^\mu(-k_2-k_3)(W^{+\nu}(k_2)W^{-\rho}(k_3)+W^{-\nu}(k_2)W^{+\rho}(k_3))\,+ \\ & + D^{ZWW}Z^\mu(-k_2-k_3)(W^{+\nu}(k_2)W^{-\rho}(k_3)+W^{-\nu}(k_2)W^{+\rho}(k_3))\,+ \\ & + D^{ZZZ}Z^\mu(-k_2-k_3)Z^\nu(k_2)Z^\rho(k_3)\,+ \\ & + D^{AZZ}A^\mu(-k_2-k_3)Z^\nu(k_2)Z^\rho(k_3)\,+ \\ & + D^{ZAZ}Z^\mu(-k_2-k_3)A^\nu(k_2)Z^\rho(k_3)\,+ \\ & + D^{AAZ}A^\mu(-k_2-k_3)A^\nu(k_2)Z^\rho(k_3)\,+ \\ & + \text{quadratic terms and higher powers}\Big\}\,, \end{aligned} \tag{5.23}$$

corresponding to the triangular diagrams. In (5.23), the D constants are determined as linear combinations of only two constants

$$D^{AWW} = \frac{\gamma}{\sqrt{1+\gamma^2}}\frac{B-L}{4}\left(\delta_R^2-(1-\delta_L)^2\right), \tag{5.24}$$

$$D^{AAZ} = \frac{\gamma^2}{\sqrt{1+\gamma^2}}\frac{B-L}{2}\left(\delta_R+\delta_L-1\right), \tag{5.25}$$

by the following relations

$$D^{ZWW} = -\gamma D^{AWW}\,, \tag{5.26}$$

$$D^{ZAZ} = -\gamma D^{AAZ}\,, \tag{5.27}$$

$$D^{AZZ} = \left(1+\gamma^2\right)D^{AWW} - \gamma D^{AAZ}\,, \tag{5.28}$$

$$D^{ZZZ} = -\gamma\left(\left(1+\gamma^2\right)D^{AWW} - \gamma D^{AAZ}\right). \tag{5.29}$$

Thus we find that these loops do in particular generate trilinear interactions between neutral electroweak vector bosons. The implications of anomalies and of the parameters δ_L, δ_R on these interactions may then be studied following [52]–[55]. Let us just point out that anomalous contributions from each doublet would vanish if[8]

$$\delta_L + \delta_R = 1\,. \tag{5.30}$$

Note that renormalizability order-by-order in the sense of Weinberg [1] is not lost: on the contrary, we have seen that the three-point function of the techni-currents obey the

[8]There is no contradiction with the fact that elementary fermions have $\delta_L = \delta_R = 0$ at this order: for elementary fermions, the absence of anomalous contributions to three-point functions of the electroweak vector bosons follows from the condition on the trace of $B-L$.

appropriate transformation properties as deduced from the techni-theory, and which serve as a guide for the construction of the effective lagrangian. On the other hand, we find that there are in general anomalous contributions to three-point functions of electroweak vector bosons. It should be stressed that there is no conflict here: the symmetry of the techni-theory on which the LEET hinges must be anomaly-free, and the generating functional must reflect this. On the other hand, the symmetries whose currents are coupled to the elementary gauge bosons may be anomalous. Since the effective theory framework is not based on the requirement of renormalizability in the usual sense, this does not endanger the consistency of the theory.

6. Conclusion

In this paper, we have investigated the minimal low-energy effective theory description of EWSB, where renormalization proceeds order-by-order in the momentum expansion instead of the usual renormalization order-by-order in powers of the coupling constants. In particular, we have analyzed the relation implied by the anomaly-matching between the effective theory operating with low-energy degrees of freedom (Goldstone bosons and, possibly, light composite fermions) and the fundamental 'techni-theory', representing the unknown ultra-violet completion of the LEET.

We have derived a necessary condition, which the techni-theory has to satisfy in order to allow all Goldstone modes to be removed from the spectrum by the Higgs mechanism: symmetries of the techni-theory under which the low-energy degrees of freedom are charged must be anomaly-free, that is $k_{\mathrm{TT}} = 0$ in the notation of section 3. This conclusion is true independently of the presence of composite fermions: the anomaly-matching allowed us to phrase the question directly in terms of the underlying techni-theory. Technicolor models are usually constructed to be anomaly-free [48]. The relevant question is then how the restriction $k_{\mathrm{TT}} = 0$ modifies the spectrum of low-lying bound states as compared to QCD in which $k_{\mathrm{TT}} = N_c/3$. Beyond such models, translating the restriction $k_{\mathrm{TT}} = 0$ in terms of the fundamental variables of a generic techni-theory is far from obvious: to achieve this, one would have to go beyond the effective theory.

Concerning the anomaly-matching, we have extended the construction of the Wess-Zumino effective lagrangian to the case where the low-energy sector contains composite fermions in addition to Goldstone bosons. This turns out to be possible whatever the values of the $\mathrm{U}(1)_{B-L}$ charges of the composite fermions, and for any values of their non-standard couplings δ_L, δ_R. Consequently, there are no restrictions on these couplings. The possibility of having an (incomplete) fourth generation of fermions, which would be composite is thus theoretically open.

Within the spurion formalism developed in [25], the distinction between composite and elementary fermions is unambiguous: since the composite and elementary sectors of the theory are coupled only via spurions, the mixing between composite and elementary fermions is an effect of higher orders in the spurion expansion. Qualitatively, the exclusive properties of composite fermions should be: i) the non-standard couplings δ_L, δ_R describing a violation of universality of left-handed couplings to electroweak vector bosons, as well as right-handed couplings to $W^{\pm}$, ii) a relatively large mean mass m of the doublet and a splitting within the doublet $\Delta m \ll m$, iii) no restriction on $B-L$, that is a possibility to have incomplete composite fermion generations. These properties practically exclude the interpretation of known fermion doublets as composite in the above sense. We have also shown in section 5 that composite fermions would indirectly manifest themselves in the triple boson vertices induced by anomalous triangular graphs. Only composite fermions that have $\delta_L + \delta_R \neq 1$ can contribute to such graphs.

Elementary fermions necessarily have $\delta_L = \delta_R = 0$ at the leading-order of the spurion expansion. They do not participate in the anomaly-matching, being exterior to the composite sector. Nevertheless, as shown in section 4, one recovers the usual condition that the trace of $B-L$ over elementary fermions vanishes. The reason for this is not the usual requirement of renormalizability but rather the assumption that all Goldstone modes are removed from the spectrum by the Higgs mechanism. The whole effective theory can then be consistently formulated in the unitary gauge.

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